

# Semidiurnal signals in UT1/LOD due to the influence of tidal gravitation on the triaxial structure of the Earth

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# Introduction

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## The problem

The axial component of Earth rotation, which is expressed by Universal Time UT1, contains small variations due to the influence of tidal gravitation on the triaxial structure of the Earth.

- Predicted quite early by, e.g., Tisserand (1891), Kinoshita (1977), but neglected in computation due to very small size (maximum peak-to-peak variation about 0.1 mas)
- More complete descriptions have been published when the effect became observationally significant
  - Chao et al., 1991, *Geophys. Res. Letters*, Vol. 18, pp. 2007–2010;
  - Wunsch, 1991, *Astron. Nachrichten*, Vol. 312, pp. 321–325;
  - Brzeziński and Capitaine, 2002, *Proc. Journees 2001*, 51–58.
- However, the model is still not included in the IERS Conventions, in contrast to the corresponding effect in polar motion (IERS Conventions 2003, Table 5.1)

## Our purpose

- Derive an analytical solution for the lunisolar perturbations in UT1 associated with the triaxiality of the Earth, assuming 2-layer model consisting of elastic mantle and liquid core
- Estimate coefficients of the model and compare them to the available alternative solutions as well as to the ocean tide influences

## Lunisolar torque on the triaxial Earth

The torque exerted on the Earth by the external point mass  $M_d$  (the Moon or the Sun) at the geocentric position  $\vec{r}_d = (r_d \sin \theta_d \cos \lambda_d, r_d \sin \theta_d \sin \lambda_d, r_d \cos \theta_d)^T$ , can be evaluated as the negative of the torque on  $M_d$  due to the Earth, which yields the following expression

$$\vec{L} = -M_d \vec{r}_d \times \nabla U(\vec{r}_d), \quad (1)$$

where  $\times$  – vector product,  $\nabla$  – gradient operator, and  $U$  denotes gravitational potential of the Earth which can be conveniently expanded in spherical harmonics

$$U(\vec{r}) = \frac{GM}{r} \sum_{\ell=0}^{\infty} \sum_{j=0}^{\ell} \left(\frac{R_e}{r}\right)^{\ell} P_{\ell j}(\cos \theta) [C_{\ell j} \cos j\lambda + S_{\ell j} \sin j\lambda], \quad (2)$$

with

$G$  – gravitational constant,

$M, R_e$  – mass and mean equatorial radius of the Earth,

$P_{\ell j}$  – associated Legendre function of degree/order  $(\ell, j)$ ,

$C_{\ell j}, S_{\ell j}$  – Stokes (un-normalized) coefficients evaluated at  $r = R_e$ .

Substitution yields the following expression for the axial torque

$$L_3 = -M_d \frac{\partial U(r_d, \theta_d, \lambda_d)}{\partial \lambda_d}. \quad (3)$$

For further derivations it is necessary to know the time dependence of  $r_d, \theta_d, \lambda_d$ .

## Lunisolar torque on the triaxial Earth

We apply the procedure developed by Woolard (1953) and described by McClure (1974). The time dependence of  $r_d, \theta_d, \lambda_d$  is derived from the analytical development the tide generating potential  $u(\vec{r})$

$$u(\vec{r}) = \frac{GM_d}{c_d} \sum_{\ell=2}^{\infty} \sum_{j=0}^{\ell} \left(\frac{r}{c_d}\right)^{\ell} P_{\ell j}(\cos \theta) \sum_s A_{\ell j s} \cos \left[ \omega_{\ell j s} t + \beta_{\ell j s} + j\lambda + (\ell - j)\frac{\pi}{2} \right], \quad (4)$$

where

$c_d$  – mean distance of the disturbing body,

$\omega_{\ell j s}, \beta_{\ell j s}$  – frequency and phase of the tidal component of degree  $\ell$ , order  $j$  and number  $s$ ,

$A_{\ell j s}$  – amplitude of the expansion.

Note:  $\omega_{\ell j s} \approx j\Omega$ , where  $\Omega$  – angular frequency of diurnal sidereal rotation. Therefore the component  $u_{\ell j}$  of the expansion (4) expresses the long periodic ( $j=0$ ), diurnal ( $j=1$ ), semidiurnal ( $j=2$ ), terdiurnal ( $j=3$ ), etc., tide.

For each term  $U_{\ell j}$ ,  $\ell \geq 2$ , of geopotential (2) we could derive explicit expressions for both the equatorial and axial torques involving certain components  $u_{\ell' j'}$  of tidal potential (4). These expressions take form of harmonic expansion with summation with respect to  $s$ . The following table gives review of all components of the lunisolar excitation having influence on Earth rotation, which exceed the level 0.1 microarcsecond ( $\mu\text{as}$ ).

# Lunisolar perturbations in Earth rotation

*Table 1:* Lunisolar perturbations in Earth rotation. Last column shows the sum of the absolute values of all amplitudes greater than  $0.01 \mu\text{as}$ . Table includes only those components of perturbation for which the total effect exceeds the level of  $0.1 \mu\text{as}$ . Note: UT1 is expressed in angular units using the correspondence  $1\mu\text{s} - 15\mu\text{as}$ .

Geo-potential	Tidal potential	Nutation	Polar motion	UT1	Maximum total amplitude (in $\mu\text{as}$ )
$U_{\ell 0}$ for $\ell = 2, 3, \dots$	$u_{\ell 1}$	long periodic	retrograde diurnal	— —	nutations $>10^7$ + precession
$U_{31}$	$u_{30}$	prograde	long	—	91.3
$U_{41}$	$u_{40}$	diurnal	periodic	—	1.0 + drift $5.7 \mu\text{as/yr}$
$U_{22}$	$u_{21}$	prograde	prograde	—	51.6
$U_{32}$	$u_{31}$	semidiurnal	diurnal	—	0.2
$U_{33}$	$u_{32}$	prograde terdiurnal	prograde semidiurnal	—	0.1
$U_{31}$	$u_{32}$	retrograde diurnal	retrograde semidiurnal	—	0.8
$U_{32}$	$u_{33}$	retrograde semidiurnal	retrograde terdiurnal	—	0.1
$U_{22}$	$u_{22}$	—	—	semi-	62.9
$U_{32}$	$u_{32}$	—	—	diurnal	0.3
$U_{31}$	$u_{31}$	—	—	diurnal	1.1
$U_{33}$	$u_{33}$	—	—	terdiurnal	0.3

## UT1 variation due to the triaxial figure of the Earth

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Consider now the special case of  $U_{22}$ . We find the following expression for the axial torque

$$L_3 = -24 \frac{GMM_d R_e^2}{c_d^3} D_{22} \sum_s A_{22s} \sin(\omega_{22s} t + \beta_{22s} + 2\lambda_{22}), \quad (5)$$

in which

$$C_{22} + iS_{22} = D_{22}(\cos 2\lambda_{22} + i \sin 2\lambda_{22}) = D_{22}e^{i2\lambda_{22}}, \text{ with } i = \sqrt{-1}.$$

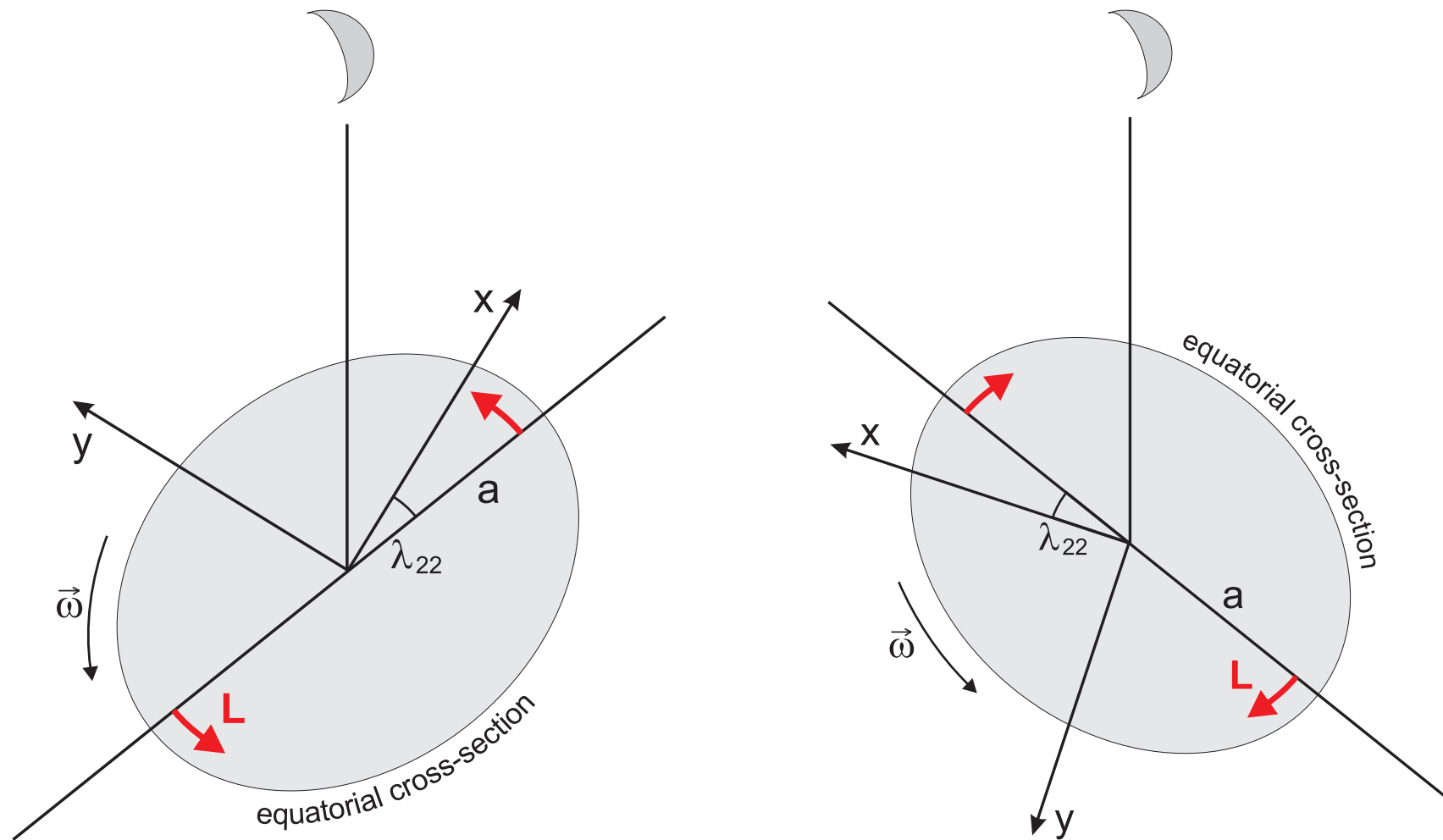
Note: if  $A < B < C$  are the principal moments of inertia of the Earth, then

- $D_{22} = (B - A)/4MR_e^2$  is proportional to the equatorial dynamical flattening
- $\lambda_{22}$  is the angle between the  $x$ -axis of the TRS and the  $A$ -principal axis of inertia

Remarks:

1. The torque is proportional to the equatorial flattening of the Earth. Parameters  $D_{22}$ ,  $\lambda_{22}$  can be derived with high precision from the satellite determinations of gravity potential.
2. The summation extends over the set of the second-degree semidiurnal tides with coefficients  $A_{22s}$ ,  $\omega_{22s}$ ,  $\beta_{22s}$  which are known from the tidal developments.
3. The physical mechanism of the effect is shown schematically in the following figure.

# UT1 variation due to the triaxial figure of the Earth



*Figure 1:* Physical mechanism of the semidiurnal UT1 variation due to the lunisolar tidal torque on the triaxial Earth. Earth rotation is accelerated when the equatorial major semi-axis approaches the line Earth–Moon (left), and decelerated after crossing this line (right). This configuration repeats after  $180^\circ$  corresponding to 12 hours.

## UT1 variation due to the triaxial figure of the Earth

Now we estimate the Earth response to the axial torque under the following assumptions:

- The liquid core does not participate in the rotational variations;
- The satellite-observed triaxiality of the Earth comes from the mantle alone;
- Contributions to  $U_{22}$  from the elastic deformations of the mantle due to the influence of the  $u_{22}$  tidal potential are neglected as a secondary effect.

The solution is

$$\Delta\text{UT1} = \frac{E \kappa}{C_m \Omega} \iint L_3 dt^2, \quad (6)$$

where

$E = 1 + \frac{4k_2 C-A}{3k_s C})^{-1} \approx 0.999$  expresses influence of elastic rotational deformations,  $\kappa = 0.997270$  is the conversion factor between the sidereal and solar time scales,  $C_m$  denotes principal axial moment of inertia of the mantle.

Substitution of the axial torque  $L_3$  expressed by eq.(5) and double integration yields

$$\Delta\text{UT1} = 24 \frac{E \kappa G M M_d R_e^2}{c_d^3 C_m \Omega^3} D_{22} \sum_s \frac{A_{22s} \sin(\omega_{22s} t + \beta_{22s} + 2\lambda_{22})}{\left(\frac{\omega_{22s}}{\Omega}\right)^2}. \quad (7)$$

By inserting  $E=1$  and replacing  $C_m$  by  $C$  we get the corresponding solution for the rigid Earth.

# UT1 variation due to the triaxial figure of the Earth

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## Computational details and results

### *Parameters of the model*

$D_{22} = 1.815528 \times 10^{-6}$ ,  $2\lambda_{22} = -29.8581^\circ$  from the geopotential expansion JGM-3

Coefficients of the TGP: Hartmann and Wenzel (1995) (after conversion to present format)

$C/C_m = 1.1283$  from model PREM

### *Argument convention*

Each tidal argument of expansion (7) is expressed below using the Woolard convention

$$\omega_{22s}t + \beta_{22s} + 2\lambda_{22} = 2(\chi + \lambda_{22}) + k_1l_m + k_2l_s + k_3F + k_4D + k_5\hat{\Omega}, \quad (8)$$

where  $\chi = \text{GMST} + \pi$  and  $l_m$ ,  $l_s$ ,  $F$ ,  $D$ ,  $\hat{\Omega}$  are the so-called “fundamental arguments” and  $k_1, \dots, k_6$  are integer coefficients identifying the tidal component.

### *Results*

Table 2 below shows coefficients of expansion (7) computed for the rigid Earth and for the model comprising the elastic mantle and the liquid core. Our estimates are compared to the results of Wünsch (1991) and Chao et al. (1991). We also shown for comparison the amplitudes of the ocean tide influences (IERS Conventions, 2003, Table 8.3b).

# UT1 variation due to the triaxial figure of the Earth

*Table 2:* Semidiurnal variations of UT1 due to the action of the lunisolar tidal potential on the triaxial Earth. The amplitudes for a rigid Earth (denoted “Rigid”) and for elastic Earth with liquid core (“2-layer”) have been computed using the TGP expansion of Hartmann and Wenzel (1995). Our results are compared to the models of Wünsch (1991) (“Wu91”), Chao et al. (1991) (“Ch91”), and to the ocean tide contributions (OTAM). Listed are all terms with amplitudes larger than  $0.5 \mu\text{as}$ . Units are  $\mu\text{as}$ ;  $\Phi$  denotes  $\text{GMST} + \pi - 14.92905^\circ$ .

Argument						Tidal component			UT1					
$\Phi$	$l_m$	$l_s$	$F$	$D$	$\hat{\Omega}$	Doodson number	Darwin symbol	Origin	Period day	Rigid sin	2-layer sin	Wu91 sin	Ch91 sin	OTAM amp.
2	-2	0	-2	0	-2	235.755	$2N_2$	M	0.5377239	0.73	0.83	0.74	—	10.0
2	0	0	-2	-2	-2	237.555	$\mu_2$	M	0.5363232	0.88	0.99	0.89	—	11.6
2	-1	0	-2	0	-2	245.655	$N_2$	M	0.5274312	5.34	6.02	5.34	4.8	61.5
2	1	0	-2	-2	-2	247.455	$\nu_2$	M	0.5260835	1.01	1.14	—	—	11.4
2	0	0	-2	0	-1	255.545	$M'_2$	M	0.5175645	-1.00	-1.13	-1.01	—	9.9
2	0	0	-2	0	-2	255.555	$M_2$	M	0.5175251	26.84	30.28	26.84	29.0	265.5
2	1	0	-2	0	-2	265.455	$L_2$	M	0.5079842	-0.73	-0.82	-0.74	—	6.6
2	0	-1	-2	2	-2	272.556	$T_2$	S	0.5006854	0.68	0.77	0.69	—	6.6
2	0	0	-2	2	-2	273.555	$S_2$	S	0.5000000	11.65	13.15	11.66	13.0	113.3
2	0	0	0	0	0	275.555	$K_2$	M	0.4986348	2.15	2.43	2.16	3.6	31.5
2	0	0	0	0	0	275.555	$K_2$	S	0.4986348	1.00	1.13	0.99	—	—
2	0	0	0	0	-1	275.565	$K'_2$	M	0.4985982	0.94	1.06	0.95	—	9.5
Sum of all amplitudes $>0.01 \mu\text{as}$ (68 terms)										55.71	62.86			

## Conclusions

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We considered in this work the lunisolar perturbations in UT1 associated with departures of the Earth's dynamical figure from the rotational symmetry. We arrived to the following conclusions:

- The only terms with amplitudes exceeding the truncation level of  $0.5 \mu\text{as}$  are 11 semidiurnal harmonics due to the influence of the second degree tidal potential  $u_{22}$  on the equatorial flattening of the Earth expressed by the Stokes coefficients  $C_{22}$ ,  $S_{22}$ .
- There is an excellent agreement between the amplitudes derived by Wunsch (1991) and our estimates assuming model of the rigid Earth. The only important difference is the term  $\nu_2$  which seems to be overlooked in the development of Wunsch.
- Accounting for the presence of the liquid core increases all amplitudes by the factor  $C/C_m = 1.128$ . Our amplitudes computed for an elastic Earth with liquid core are in reasonable agreement with those derived by Chao et al. (1991), but the last model was not complete.
- Estimated effect is superimposed on the ocean tide influence having the same frequencies but 9 to 11 times larger amplitudes. Nevertheless, its maximum peak-to-peak size is about  $7 \mu\text{s}$  ( $105 \mu\text{as}$ ) hence definitely above the current uncertainty of UT1.
- We recommend adding this model to the set of procedures provided by the IERS Conventions. Comparison with the model of prograde diurnal polar motion due to the triaxiality (IERS Conventions, Tab. 5.1) shows that: 1) the two effects are of similar size, 2) there is consistency between the underlying dynamical models, parameters employed, etc.