Investigating ultra-long gravitational waves with measurements of pulsars rotational parameters

Maxim Pshirkov,
PRAO ASC LPI,
Russia

XXVII IAU GENERAL ASSEMBLY, Rio de Janeiro,
Joint Discussion 6 “Time and Astronomy”, 7 Aug 2009

Introduction

- Search for gravitational waves (GW) is “Holy Grail” of modern physics

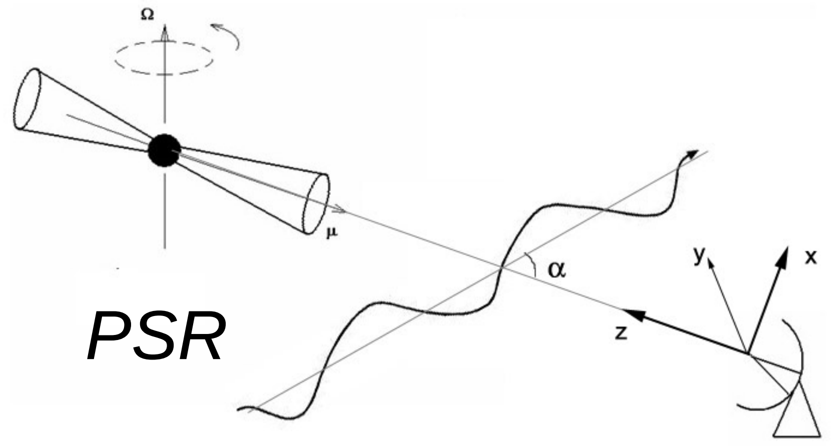
- Every implemented technique has its own frequency limitations:
  - Ground-based interferometers: 10-1000 Hz
  - Space-borne interferometers (future): 10^{-5}-1 Hz
  - Pulsar timing: 10^{-9} -10^{-7} Hz
  - CMB: 10^{-18} – 10^{-16} Hz

- Our goal: extend frequency range in ultra-low region using existing pulsar data

Pulsar timing and gravitational waves detection: basics (1/2)

•GW interacts with electromagnetic waves from the pulsar affecting apparent frequency (Sazhin,1978; Detweiler, 1979)

$$\frac{\Delta \nu}{\nu} = -\frac{1}{2}\int \frac{\partial h_{zz}}{\partial t} dl$$

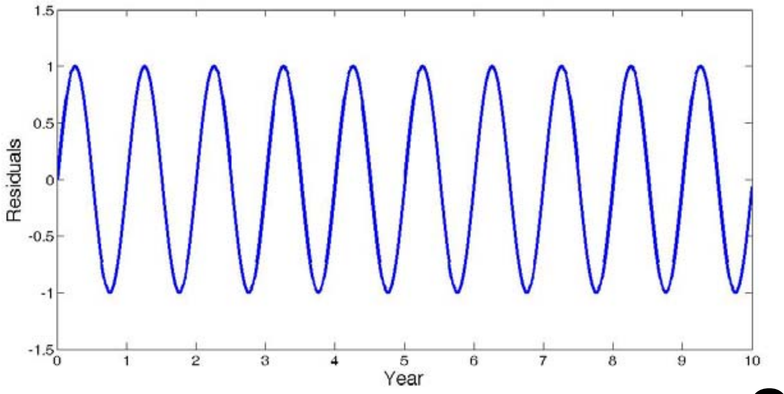


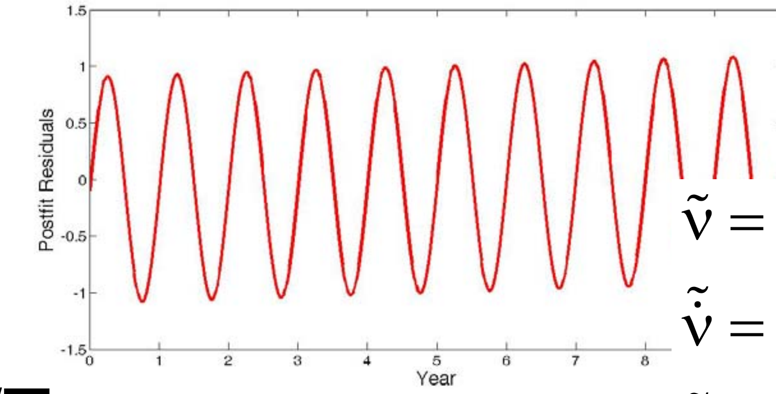
•These variations will show up in anomalous timing residuals:

$$R(t) = \int_0^t \left[\nu_0 - \nu(t)\right] / \nu_0 dt \sim \frac{h_0}{\omega}\cos(\omega t)$$

Pulsar timing and gravitational waves detection: basics (2/2)

Prefit residuals　　　　Postfit residuals



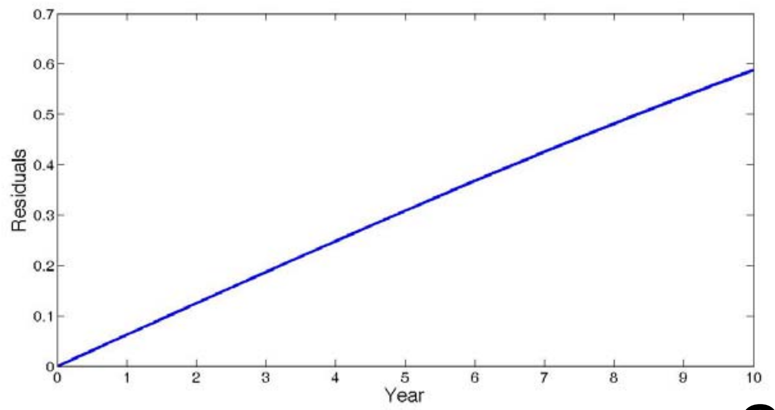


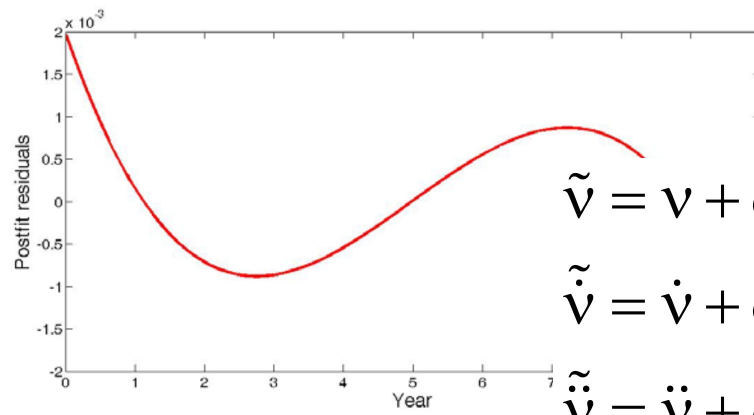
$\tilde{\nu} = \nu$

$\tilde{\dot{\nu}} = \dot{\nu}$

$\tilde{\ddot{\nu}} = \ddot{\nu}$

$\omega > 2\pi/T_{span}$





$\tilde{\nu} = \nu + \delta\nu$

$\tilde{\dot{\nu}} = \dot{\nu} + \delta\dot{\nu}$

$\tilde{\ddot{\nu}} = \ddot{\nu} + \delta\ddot{\nu}$

$\omega < 2\pi/T_{span}$

…a bit of mathematics(1/4):

GW metric:

$$h_{ij} = hp_{ij}e^{ik_\mu x_\mu} = hp_{ij}e^{-i(k_0 ct - k_i x^i)}$$

Induced variation of rotational frequency:

$$\frac{\delta\nu(t)}{\nu_0} = \frac{1}{2} he^i e^j p_{ij} e^{-ikct} \left[\frac{1 - e^{i\left(1 - \tilde{k}_i e^i\right)kD}}{1 - \tilde{k}_i e^i} \right]$$

Variations of frequency derivatives:

$$\frac{\delta\dot{\nu}(t)}{\nu_0} = \frac{-ikc}{2} he^i e^j p_{ij} e^{-ikct} \left[\frac{1 - e^{i\left(1 - \tilde{k}_i e^i\right)kD}}{1 - \tilde{k}_i e^i} \right]$$

$$\frac{\delta\ddot{\nu}(t)}{\nu_0} = \frac{-k^2 c^2}{2} he^i e^j p_{ij} e^{-ikct} \left[\frac{1 - e^{i\left(1 - \tilde{k}_i e^i\right)kD}}{1 - \tilde{k}_i e^i} \right]$$

…a bit of mathematics(2/4):

Stochastic GWB: $h_{ij}(t,x^i)=\int d^3k \sum_{s=1,2}\left[h_s(k^i,t)\overset{s}{p}_{ij}(k^i)e^{ik_ix^i}+c.c\right]$

$$\left\langle h_s(k^i)\right\rangle=0 \qquad \left\langle h_s(k^i)h^*_{s'}(k'^i)\right\rangle=\frac{P_h(k)}{16\pi k^3}\delta_{ss'}\delta^3(k^i-k'^i)$$

Response:
$$\frac{\delta\frac{d^{1,2}\nu}{dt^{1,2}}}{\nu_0}=\int d^3k\sum_{s=1,2}[h_s(k^i)\tilde{R}_{1,2}(t;k^i,s)+c.c.]$$

$$\tilde{R}_1(t;k^i,s)=\frac{-ikc}{2}e^ie^jp_{ij}e^{-ikct}\left[\frac{1-e^{i\left(1-\tilde{k}_ie^i\right)kD}}{1-\tilde{k}_ie^i}\right]$$

$$\tilde{R}_2(t;k^i,s)=\frac{-k^2c^2}{2}e^ie^jp_{ij}e^{-ikct}\left[\frac{1-e^{i\left(1-\tilde{k}_ie^i\right)kD}}{1-\tilde{k}_ie^i}\right]$$

…a bit of mathematics(3/4):

Statistical properties of response:

$$\left\langle \frac{\delta\dot{\nu}(t)}{\nu_0} \right\rangle = 0 \qquad \left\langle \left(\frac{\delta\dot{\nu}(t)}{\nu_0} \right)^2 \right\rangle = \int \frac{dk}{k} P_h(k) \tilde{R}_1^2(k)$$

$$\left\langle \frac{\delta\ddot{\nu}(t)}{\nu_0} \right\rangle = 0 \qquad \left\langle \left(\frac{\delta\ddot{\nu}(t)}{\nu_0} \right)^2 \right\rangle = \int \frac{dk}{k} P_h(k) \tilde{R}_2^2(k)$$

Transfer functions:

$$\tilde{R}_{1,2}^2(k) = \frac{1}{8\pi} \int d\Omega \sum_s \left| \tilde{R}_{1,2}(t; k^i, s) \right|^2$$

$$\tilde{R}_1^2(k) = \frac{k^2 c^2}{6} - \frac{c^2}{4D^2} + \frac{\cos(kD)\sin(kD)c^2}{4kD^3}$$

$$\tilde{R}_2^2(k) = \frac{k^4 c^4}{6} - \frac{k^2 c^4}{4D^2} + \frac{\cos(kD)\sin(kD)kc^4}{4D^3}$$

…a bit of mathematics(4/4):

Energy density per unit logarithmic interval of frequency, Ω:

$$\Omega_{gw}(k) = \frac{1}{\rho_c}\frac{d\rho_{gw}}{d\log k} = \frac{k^2c^2}{6H_0^2}P_h(k)$$

$$\Omega_{gw}(k) = \Omega_{gw}(k_0)\left(\frac{k}{k_0}\right)^{n_T}$$

Finally, combining from bits no.1-4*:

$$\left\langle\left(\frac{\delta\dot{\nu}(t)}{\nu_0}\right)^2\right\rangle = \begin{cases} \frac{c^2}{4\pi^2 n_T}\Omega_{gw}(k_0)k_H^2 k_0^{-n_T}\left[k_{\max}^{n_T} - k_{\min}^{n_T}\right], n_T \neq 0 \\ \frac{c^2}{4\pi^2 n_T}\Omega_{gw}k_H^2 \ln\left(\frac{k_{\max}}{k_{\min}}\right), n_T = 0 \end{cases}$$

$$\left\langle\left(\frac{\delta\ddot{\nu}(t)}{\nu_0}\right)^2\right\rangle = \begin{cases} \frac{c^4}{4\pi^2 (n_T+2)}\Omega_{gw}(k_0)k_H^2 k_0^{-n_T}\left[k_{\max}^{n_T+2} - k_{\min}^{n_T+2}\right], n_T \neq -2 \\ \frac{c^4}{4\pi^2 n_T}\Omega_{gw}k_0^2 k_H^2 \ln\left(\frac{k_{\max}}{k_{\min}}\right), n_T = 2 \end{cases}$$

Constraints from $\ddot{\nu}$ (1/2)

•Second derivative is more suitable for our purposes (there is *a priori* unknown spin-down term in the first derivative).

•There are a lot (~20) of pulsars with low value of $\left|\frac{\ddot{\nu}}{\nu}\right| \sim 10^{-28.5}$ (both MSP and ordinary)

•We can assume that magnitude of $\ddot{\nu}$ due to GW cannot exceed that value, thus limiting the energy density of GWB in ultra low-frequency range:

$$\Omega_{gw}(f_0) \le \frac{(n_T + 2)}{4\pi^2 H_0^2} \frac{T_{obs}^{n_T+2}}{f_0^{n_T}} \left(\frac{\ddot{\nu}_{obs}}{\nu} \right)^2$$

Constraints from $\ddot{\nu}$ (2/2)

•We can make estimations using (e.g.) properties of PSR B1937+21 (from the ATNF Pulsar Database):

$$\nu = 642 \quad \text{Hz}$$

$$\ddot{\nu} = 4\times 10^{-26}\,\text{s}^{-3}$$

$$\frac{\ddot{\nu}}{\nu} = 6.2\times 10^{-29}\,\text{s}^{-2}$$

Alternative:
implement stability parameter Δ_8 (Arzoumanian et al., 1994);
The best results come from J0437-4715.

n_T	$\Omega(f_0^*)$
0	$3\text{x}10^{-6}$
-1/2	$7\text{x}10^{-6}$
-2/3	$9\text{x}10^{-6}$
-1	$1.4\text{x}10^{-5}$

$*f_0 = 10^{-2}$ yr^{-1}

Detection (1/3)

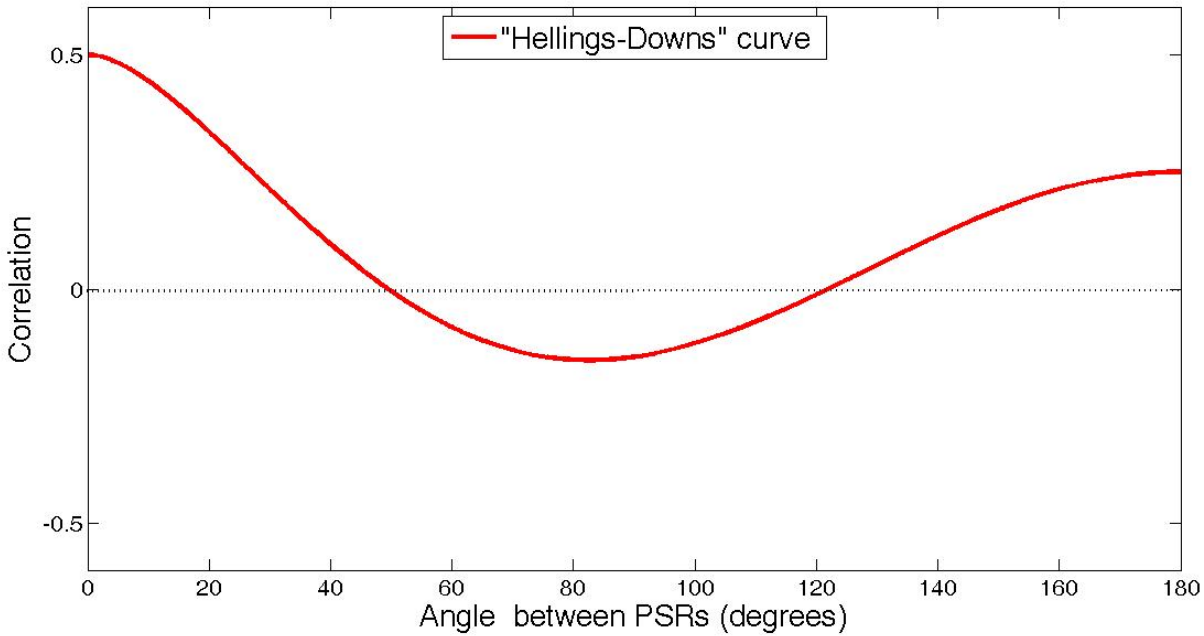
- Previous method can only place more or less stringent limits, it can not provide detection of GWB.
- For the latter, we should find unique correlation in time series of some rotational parameter for several pulsars.
- We should restrain ourselves to time series of ν and $\dot{\nu}$.
- Data are prepared as follows: total span of timing observations is sampled into shorter sub-intervals, e.g. one year long; frequency is calculated for each interval.
- The correlation coefficient between the observed values of $\Delta\nu$ (with subtraction of constant linear term to correct spin-down effect).

$$f(\theta) = \frac{1}{N} \sum_{i=0}^{i=N-1} \frac{\Delta\nu_1(t_i, e_1)}{\nu_{01}} \frac{\Delta\nu_2(t_i, e_2)}{\nu_{02}}$$

•Due to the linearity of the problem, correlation is given by usual formula:

$$\langle f(\theta) \rangle = \sigma_{\Delta\nu}^2 \zeta(\theta)$$

$$\zeta(\theta) = \frac{3(1-\cos(\theta))}{4} \log \frac{1-\cos(\theta)}{2} - \frac{1-\cos(\theta)}{8} + \frac{1}{2} + \frac{1}{2}\delta(\theta)$$



Detection (3/3)

•Correlation strength $\sigma^2_{\Delta\nu}$ can be very roughly estimated as follows:

$$\sigma^2_{\Delta\nu} = \left\langle \left(\frac{\Delta\nu}{\nu_0} \right)^2 \right\rangle \approx \left(\frac{1}{2} \frac{\delta\ddot{\nu}(t)}{\nu_0} \left(\frac{T_{samp}}{2} \right)^2 \right)^2$$

•GWB-induced part of the second derivative was estimated earlier on. Also, we can detect fractional deviations of frequency $\Delta\nu / \nu$ at $R \sim 10^{-14}$ level (rms of residuals: 1 μs, 1 year of observations). Combining that, we arrive at (assuming flat spectrum of GW):

$$\Omega_{gw} < \frac{32}{\pi^2} R^2 \left(\frac{T_H}{T_{samp}} \right)^2$$

$$\Omega_{gw} \approx 10^{-7}$$

- Influence of ultra-low frequency (10^{-12} –10^{-9} Hz) GWB can be sought in rotational parameters of PSRs.

- Precise measurement of the second derivative for >20 PSRs provide us with following constraints (depending on spectral index of GWB):

$$\Omega_{gw} \leq 10^{-6} - 10^{-5}$$

- We can use time series of rotational frequency of different pulsars to search for correlation due to the presence of GWB and thus detect that GWB.

$$\Omega_{gw} \simeq 10^{-7}$$

- **Problem calls for further, much more rigorous development!**

Acknowledgements

- That work was supported by RFBR grants no.
  - 06-02-16816-a
  - 07-02-01034-a
  - 09-02-00922-a

THANK YOU!