Cassini States of the Galilean satellites

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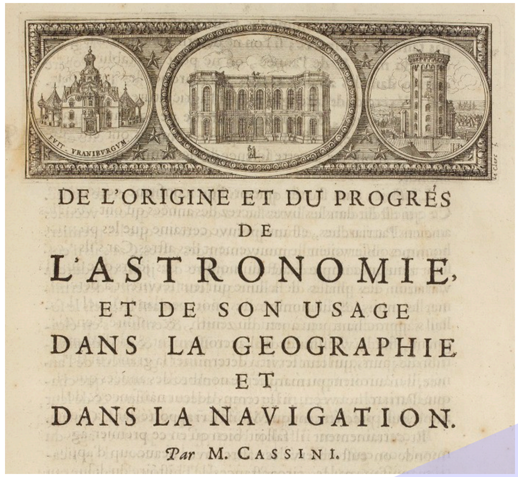
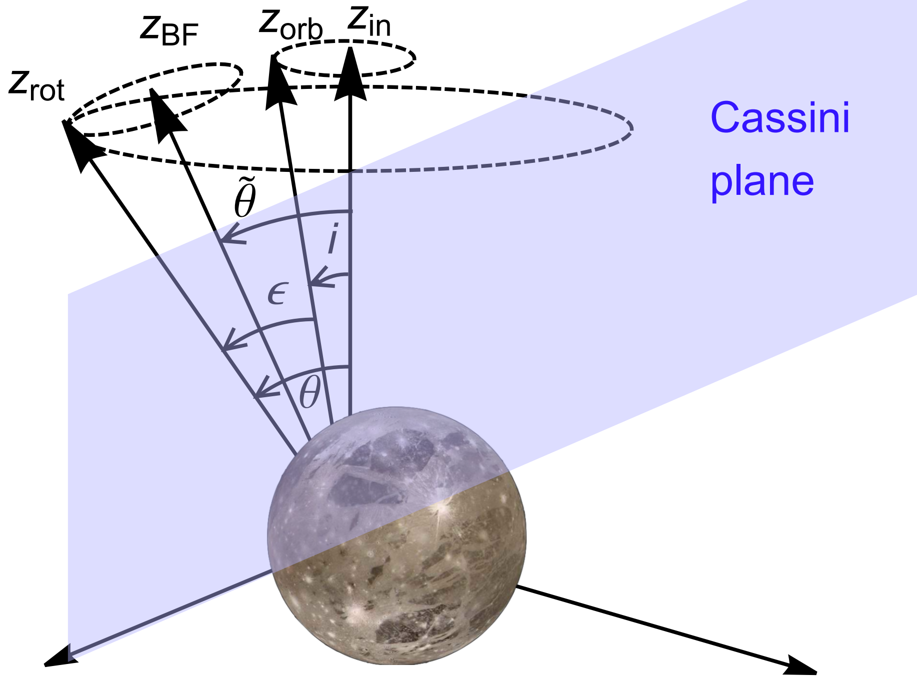
Cassini States - Definition


DE L'ORIGINE ET DU PROGRÉS
DE
L'ASTRONOMIE,
ET DE SON USAGE
DANS LA GEOGRAPHIE
ET
DANS LA NAVIGATION.
Par M. CASSINI.

Cassini's laws were established for the Moon by Cassini (1693):

1. a 1:1 spin-orbit resonance
2. precession rates of the rotation axis and of the normal to the orbit are nearly equal
3. The spin axis, the normal to the orbit and the normal to the inertial plane remain coplanar with a constant obliquity angle θ.



Cassini States - Biaxial solid case

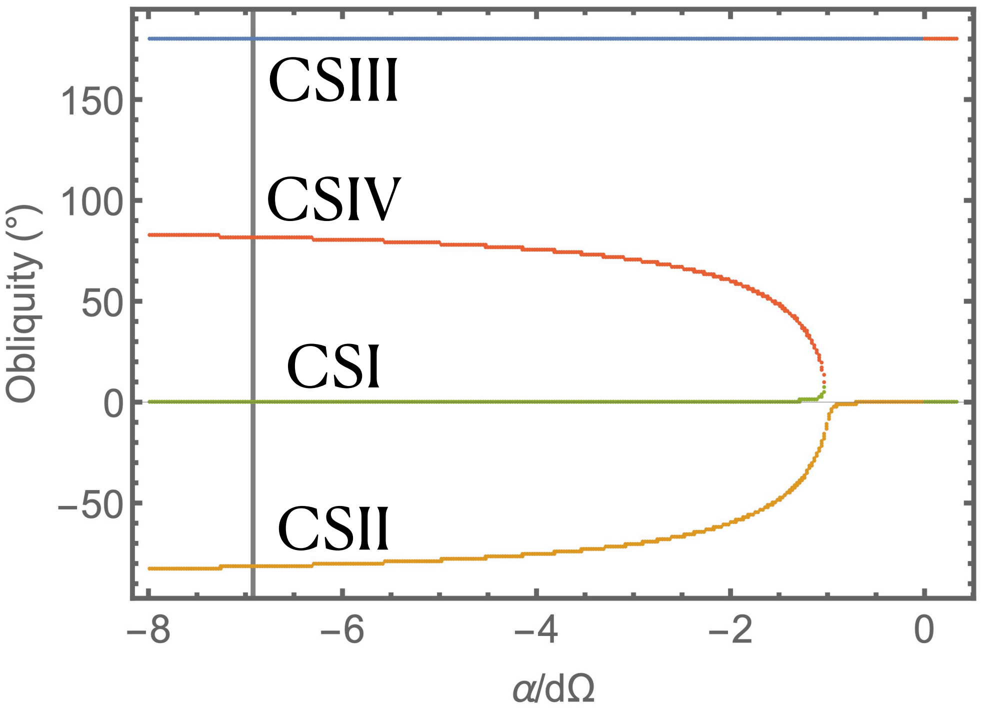
	Ganymede	Callisto
$\tilde{\theta}_{\mathrm{I}}^{\mathrm{Cl}}$ (°)	0.23763	0.62358
$\tilde{\theta}_{\mathrm{II}}^{\mathrm{Cl}}$ (°)	-76.54326	-56.08854
$\tilde{\theta}_{\mathrm{III}}^{\mathrm{Cl}}$ (°)	180.14823	180.16209
$\tilde{\theta}_{\mathrm{IV}}^{\mathrm{Cl}}$ (°)	76.90842	56.60315

For a biaxial satellite (Colombo, 1966)

$$\alpha \sin[2(\tilde{\theta} - i)] + 2\dot{\Omega} \sin \tilde{\theta} = 0,$$

where $\alpha = \frac{3}{2}\frac{C - \overline{A}}{C}n$ is the Free Precession frequency.

Ganymede



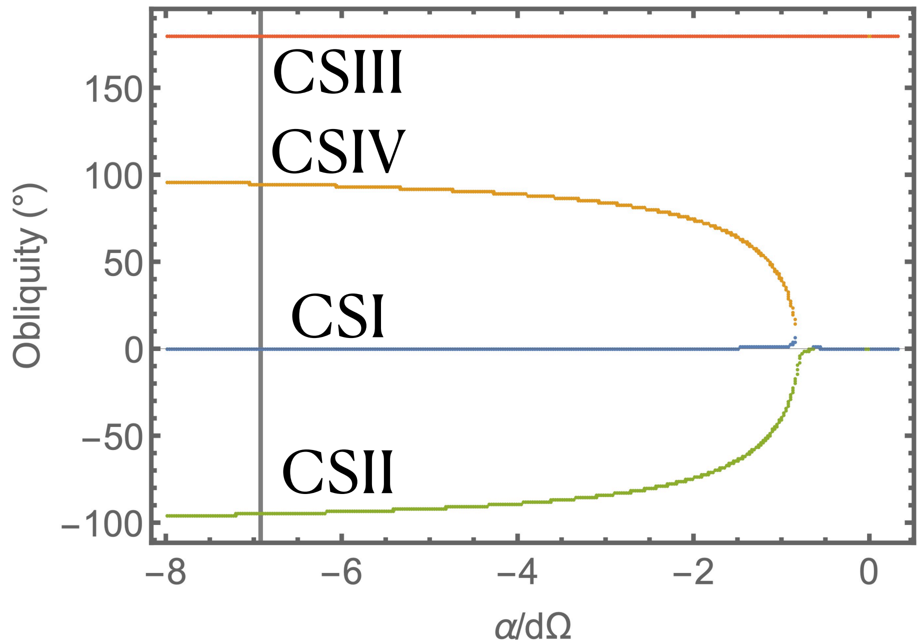
Cassini States - Triaxial solid case

	Ganymede	Callisto
$\tilde{\theta}_{\mathrm{I}}^{\mathrm{Cl,tri}}$ (°)	0.21349	0.40312
$\tilde{\theta}_{\mathrm{II}}^{\mathrm{Cl,tri}}$ (°)	-92.82688	-77.72611
$\tilde{\theta}_{\mathrm{III}}^{\mathrm{Cl,tri}}$ (°)	180.14823	180.16209
$\tilde{\theta}_{\mathrm{IV}}^{\mathrm{Cl,tri}}$ (°)	93.19646	78.21502

For a triaxial satellite (Peale 1969, Ward 1975, Henrard & Murigande, 1987, Henrard & Schwanen, 2004):

$$\frac{3}{4}\frac{B-A}{C}n\sin(\tilde{\theta}-i)+\frac{3}{8}n\frac{(4C-3A-B)}{C}\sin[2(\tilde{\theta}-i)]+2\dot{\Omega}\sin\tilde{\theta}=0$$

Ganymede

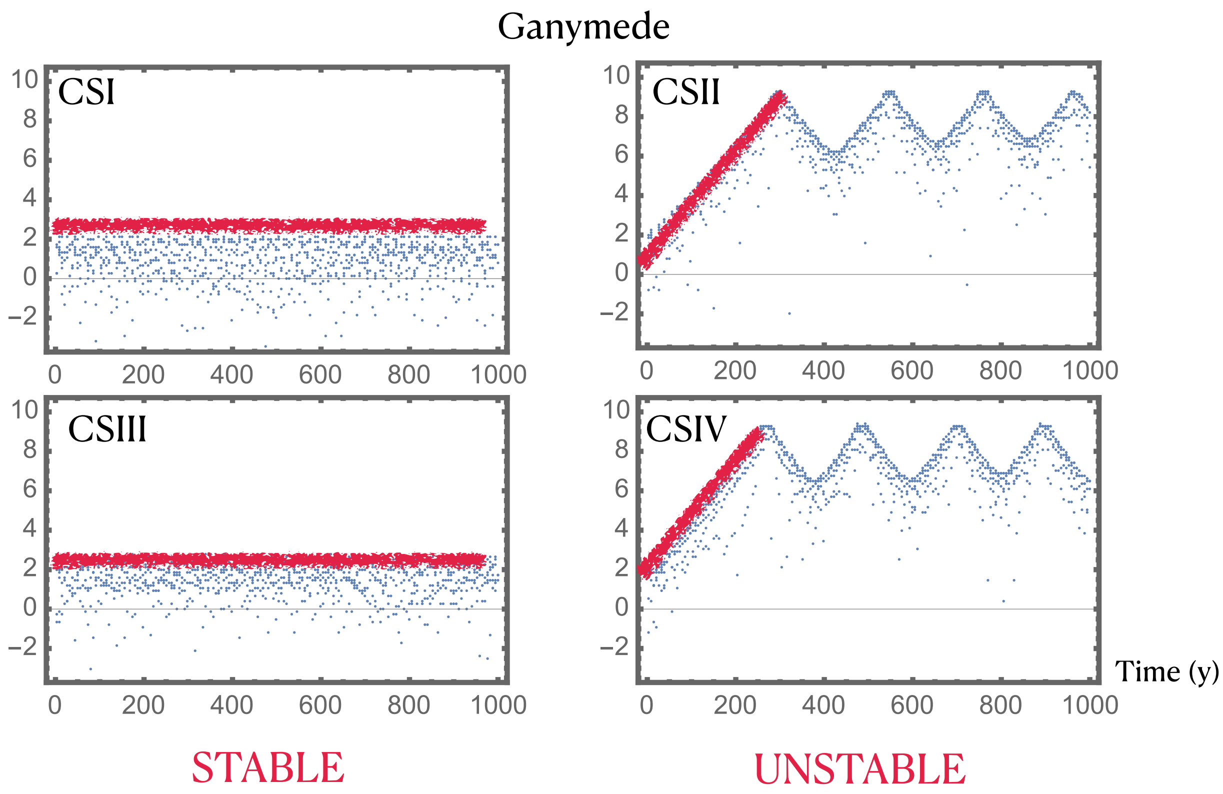


Cassini States - Stability

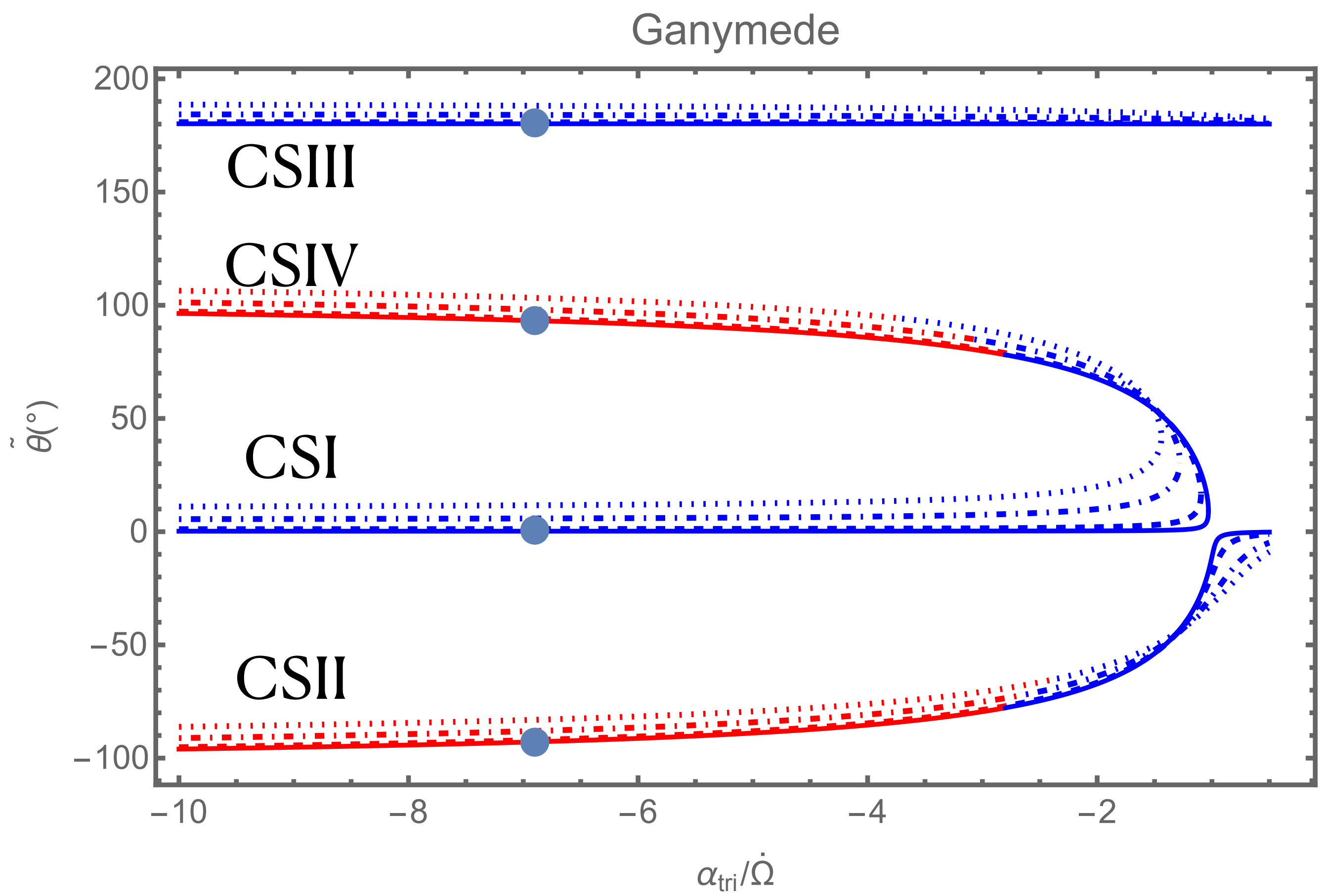
- Close to $\pm\frac{\pi}{2}$, CS II and CSIV are not stable (Henrard & Schwanen, 2004)

- Fast Lyapunov Indicator:

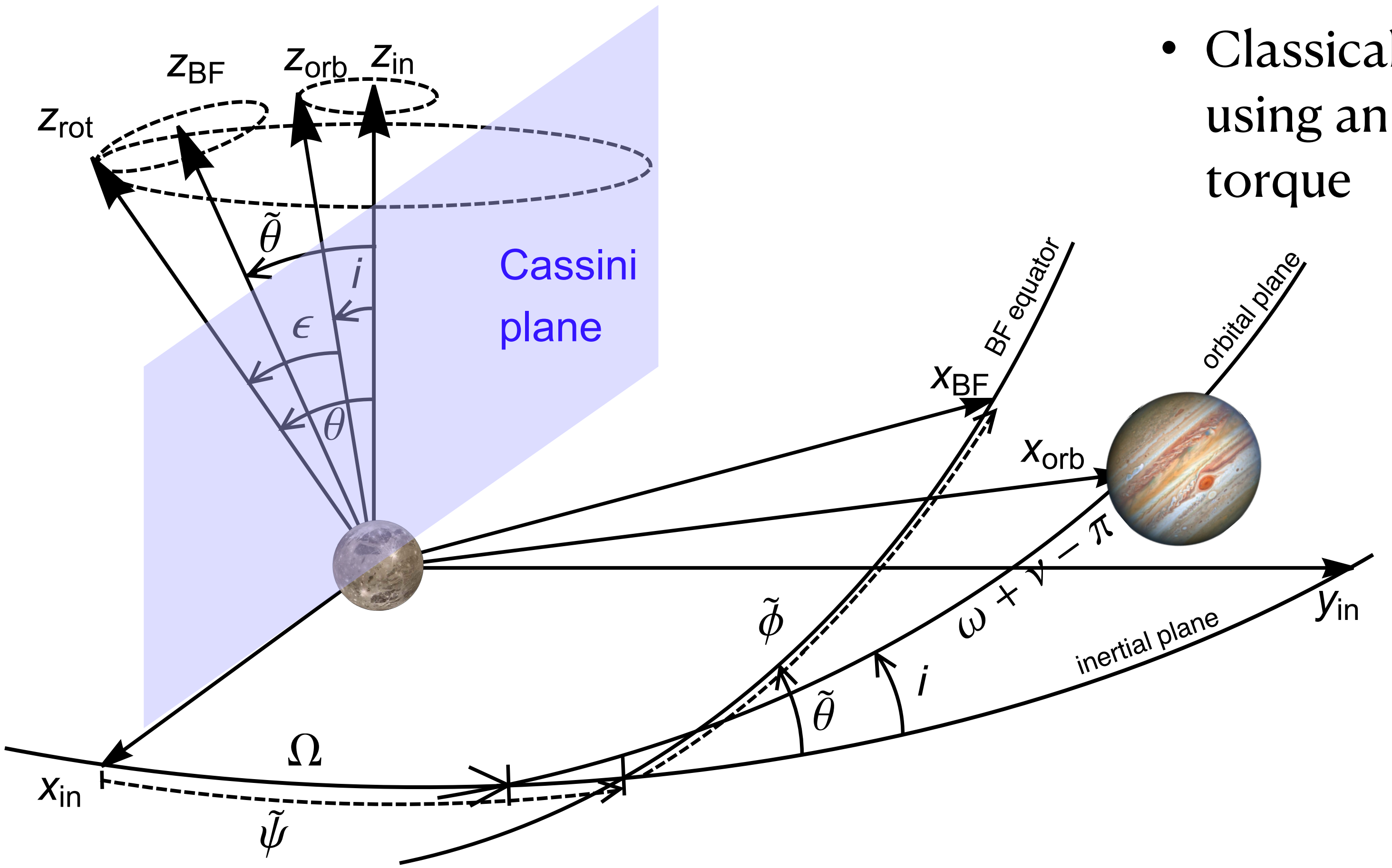
$$\mathrm{FLI}(\tilde{\theta}(0), \tilde{\theta}'(0), t) = \log \frac{|\tilde{\theta}'(k)|}{|\tilde{\theta}'(0)|}$$



Cassini States - Stability



Cassini States - A dynamical model



- Classical equations are obtained using an averaged gravitational torque
- Nutations in obliquity and in longitude ?
- Coupling between the polar motion and the obliquity ?

Angular momentum model (order 1)

Governing equations in the BF (Eckhardt, 1981) : $\frac{d\vec{H}}{dt} + \vec{\Omega} \times \vec{H} = \vec{\Gamma}$ and $\frac{d\hat{p}}{dt} + \vec{\Omega} \times \hat{p} = 0,$

For CSI, $\hat{p}(t) = (p_X, p_Y, p_Z) \simeq (0,0,1)$ and we have (order 1)

$$\begin{cases} (C-B)n^2 m_Y + Anm'_X = 0 \\ (A-C)n^2 m_X + Bnm'_Y = -3n^2(A-C)\left[p_X + i\sin(M+\omega)\right] \\ -np_Y + nm_Y + p'_X = 0 \\ np_X - nm_X + p'_Y = 0 \end{cases}$$

→ **Analytical solutions** for the order 1 polar motion components $m_i(t)$ and normal to the Laplace plane components $p_i(t)$.

Angular momentum model (order 1)

Forced solutions are given by (in the BF)

$$\begin{cases} m_X(t) = 0 \\ m_Y(t) = -\dfrac{2\dot{\Omega}}{n}\theta_0 \cos(M+\omega) \\ p_X(t) = -\dfrac{n-2\dot{\Omega}}{n}\theta_0 \sin(M+\omega) \\ p_Y(t) = -\dfrac{n-\dot{\Omega}}{n}\theta_0 \cos(M+\omega) \end{cases}$$

where (Baland et al., 2019) $\theta_0 = -\dfrac{3\frac{C-A}{B} i n^4}{\left(f^2 - \sigma^2_{\mathrm{QDFW,I}}\right)\left(f^2 - \sigma^2_{\mathrm{CW,I}}\right)} \simeq \tilde{\theta}_{\mathrm{I}}^{\mathrm{Cl}} = \dfrac{i\alpha}{\alpha + \dot{\Omega}}$

The inertial obliquity is given by $\theta_{\mathrm{I}}(t) \simeq \sqrt{\left[p_X(t) - m_X(t)\right]^2 + \left[p_Y(t) - m_Y(t)\right]^2}$ and the inertial BF obliquity by $\tilde{\theta}_{\mathrm{I}}(t) \simeq \sqrt{p_X(t)^2 + p_Y(t)^2}$.

Angular momentum model (order 2)

Governing equations in the BF (Eckhardt, 1981) : $\dfrac{d\overrightarrow{H}}{dt} + \overrightarrow{\Omega} \times \overrightarrow{H} = \overrightarrow{\Gamma}$ and $\dfrac{d\hat{p}}{dt} + \overrightarrow{\Omega} \times \hat{p} = 0,$

For CSI, we have (order 2 terms)

$$
\begin{cases}
(C-B)n^2(m_Y\gamma'/n + \delta m_Y) + An\delta m'_X = -\,3n^2(B-C)\,(\gamma - 2e\sin M)\ \left[p_X + i\sin(M+\omega)\right] \\
(A-C)n^2(m_X\gamma'/n + \delta m_X) + Bn\delta m'_Y = -\dfrac{3}{2}n^2(A-C)\big[2\delta p_X - 2p_Y\gamma + e(6p_X\cos M + 4p_Y\sin M + i\sin\omega + 5i\sin(2M+\omega) \\
-n(p_Y\gamma'/n + \delta p_Y) + n\delta m_Y + \delta p'_X = 0 \\
n(p_X\gamma'/n + \delta p_X) - n\delta m_X + \delta p'_Y = 0
\end{cases}
$$

→ **Analytical solutions** for the order 2 polar motion components $\delta m_i(t)$ and normal to the Laplace plane components $\delta p_i(t)$.

Angular momentum model (order 2)

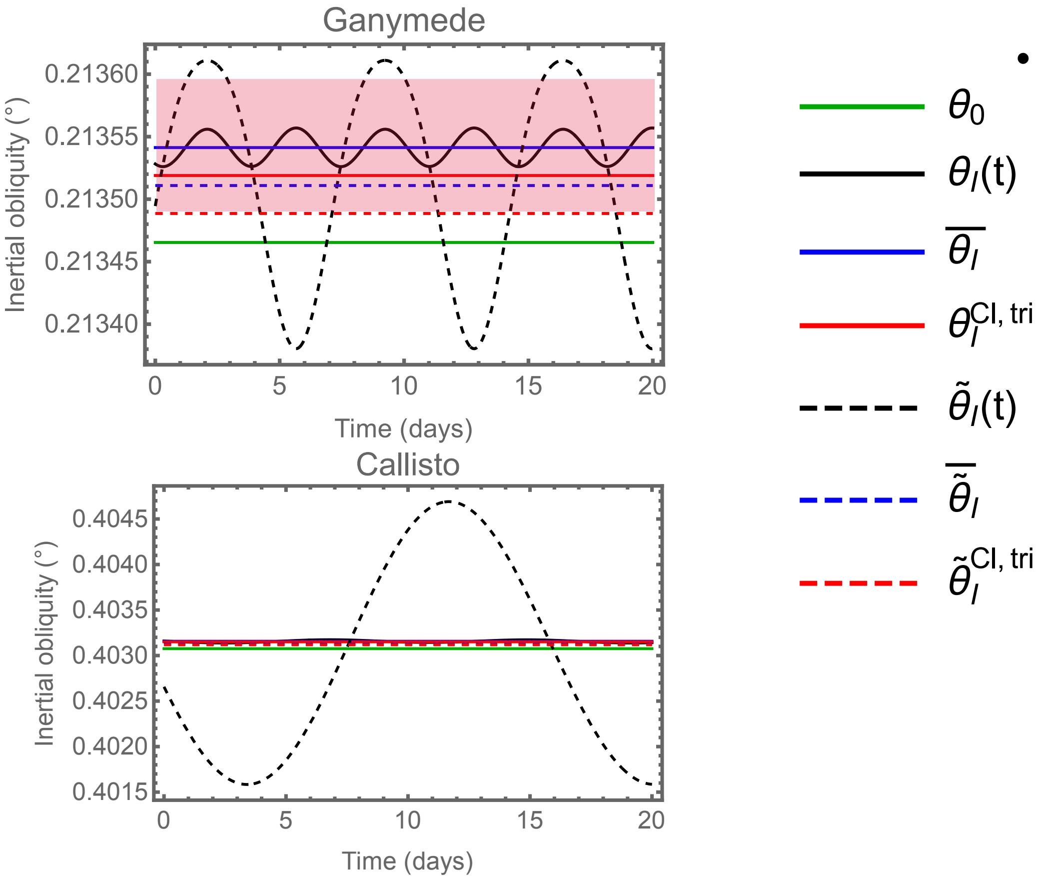
Forced solutions are given by (in the BF)

$$\begin{cases} m_X(t) = 0 + \left[\delta m_X^{e\varepsilon_0}(t) + \delta m_X^{g\varepsilon_0}(t) + \delta m_X^{gi}(t)\right] \sin\omega \\ m_Y(t) = -\dfrac{2\dot{\Omega}}{n}\theta_0 \cos(M+\omega) + \left[\delta m_Y^{e\varepsilon_0}(t) + \delta m_Y^{g\varepsilon_0}(t) + \delta m_Y^{gi}(t)\right] \cos\omega \\ p_X(t) = -\dfrac{n-2\dot{\Omega}}{n}\theta_0 \sin(M+\omega) + \left[\delta p_X^{e\varepsilon_0}(t) + \delta p_X^{g\varepsilon_0}(t) + \delta p_X^{gi}(t)\right] \sin\omega \\ p_Y(t) = -\dfrac{n-\dot{\Omega}}{n}\theta_0 \cos(M+\omega) + \left[\delta p_Y^{e\varepsilon_0}(t) + \delta p_Y^{g\varepsilon_0}(t) + \delta p_Y^{gi}(t)\right] \cos\omega \end{cases}$$

where (Baland et al., 2019) $\theta_0 = -\dfrac{3\frac{C-A}{B} i n^4}{\left(f^2 - \sigma^2_{\mathrm{QDFW,I}}\right)\left(f^2 - \sigma^2_{\mathrm{CW,I}}\right)} \simeq \tilde{\theta}_{\mathrm{I}}^{\mathrm{Cl}} = \dfrac{i\alpha}{\alpha + \dot{\Omega}}$

The inertial obliquity is given by $\theta_{\mathrm{I}}(t) \simeq \sqrt{\left[p_X(t) - m_X(t)\right]^2 + \left[p_Y(t) - m_Y(t)\right]^2}$ and the inertial BF obliquity by $\tilde{\theta}_{\mathrm{I}}(t) \simeq \sqrt{p_X(t)^2 + p_Y(t)^2}$.

Obliquity of Ganymede and Callisto (CSI)



- We obtain

Mean obliquity

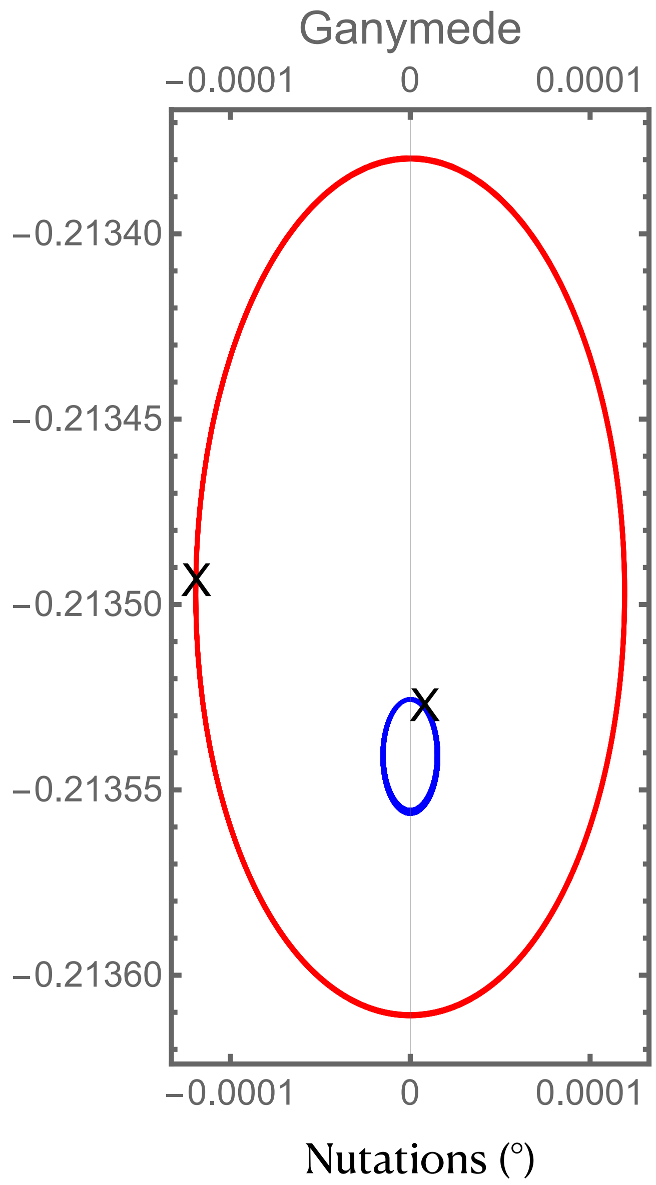
Nutations in obliquity

$$\theta_I(t) = \theta_0\left(1 - \frac{5}{2}\frac{\dot{\Omega}}{n}\right) - \frac{\theta_0}{2}\frac{\dot{\Omega}}{n}\cos\left[2(M+\omega)\right] + \frac{1}{2}(\delta M_X - \delta P_X + \delta M_Y - \delta P_Y)\cos M$$

$$\tilde{\theta}_I(t) = \theta_0\left(1 - \frac{3}{2}\frac{\dot{\Omega}}{n}\right) + \frac{\theta_0}{2}\frac{\dot{\Omega}}{n}\cos\left[2(M+\omega)\right] - \frac{1}{2}(\delta P_X + \delta P_Y)\cos M + \frac{1}{2}(\delta P_X - \delta P_Y)\cos(M+2\omega)$$

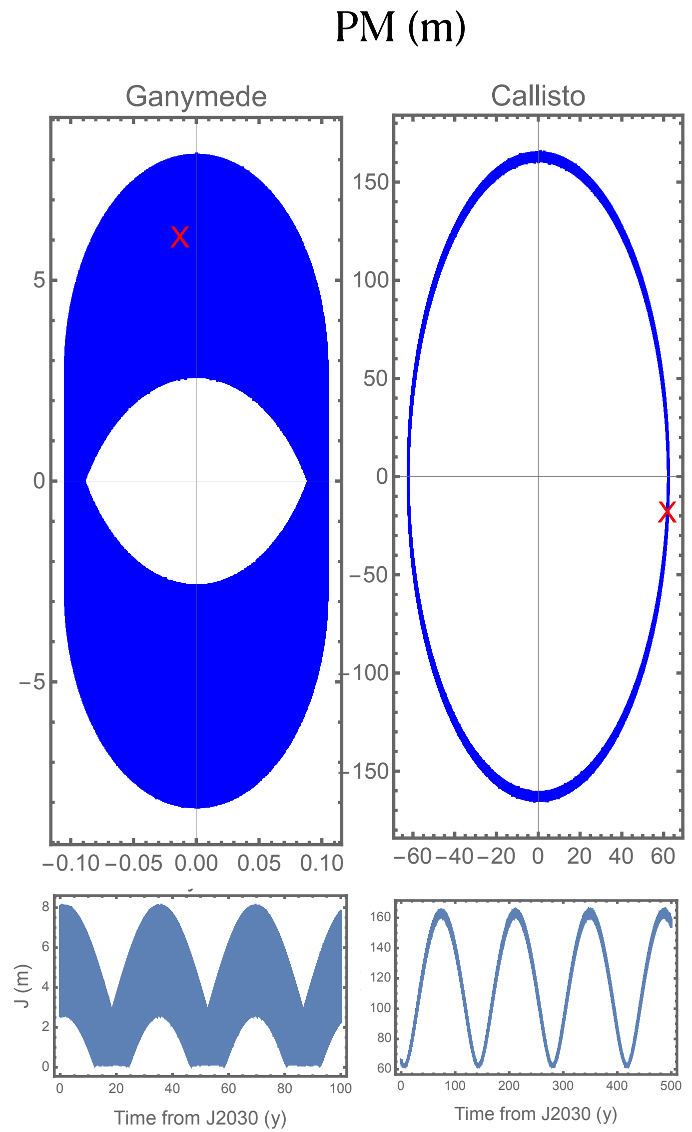
Nutations

- Anticlockwise motion of the z-axis of the BF (red) and of the spin axis (blue)



Polar motion

- Diurnal PM in y-direction
- Elliptical PM at pericenter precession period
- For Callisto, second order terms dominate over first order terms

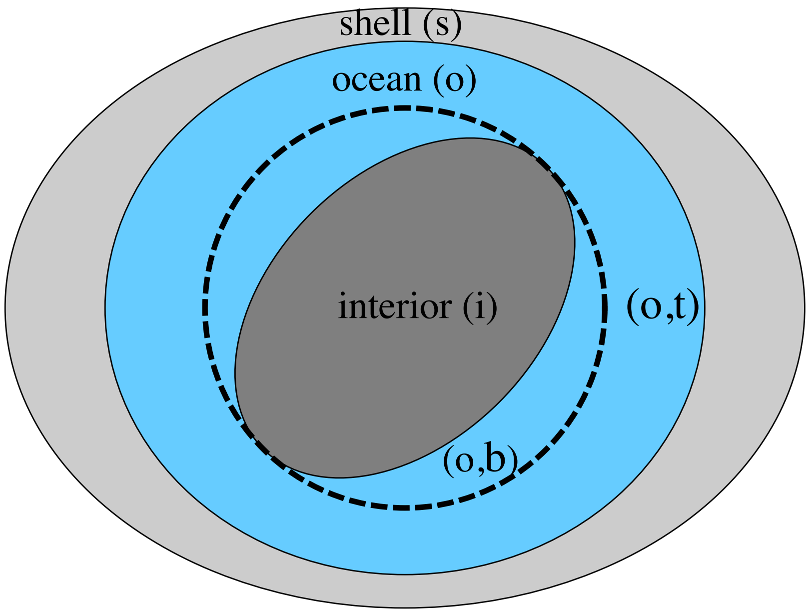


Influence of a subsurface ocean

- Global subsurface ocean (Khurana et al., 1998, Kivelson et al., 1998-2000)

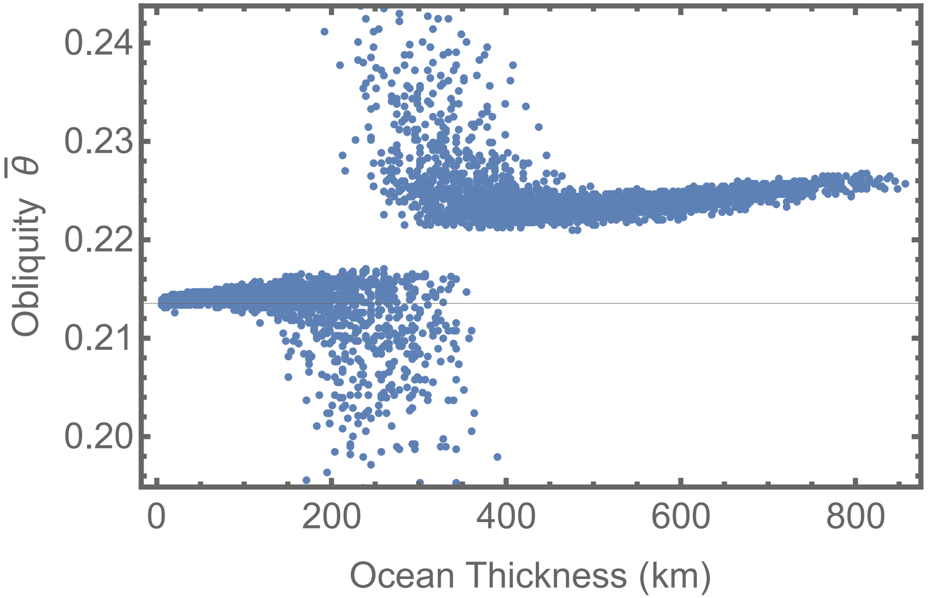
$\frac{d}{dt}\overrightarrow{H}_k + \overrightarrow{\Omega}_k \times \overrightarrow{H}_k = \overrightarrow{\Gamma}_k$ for each layer ($k = i$, $k = o$ or $k = s$)

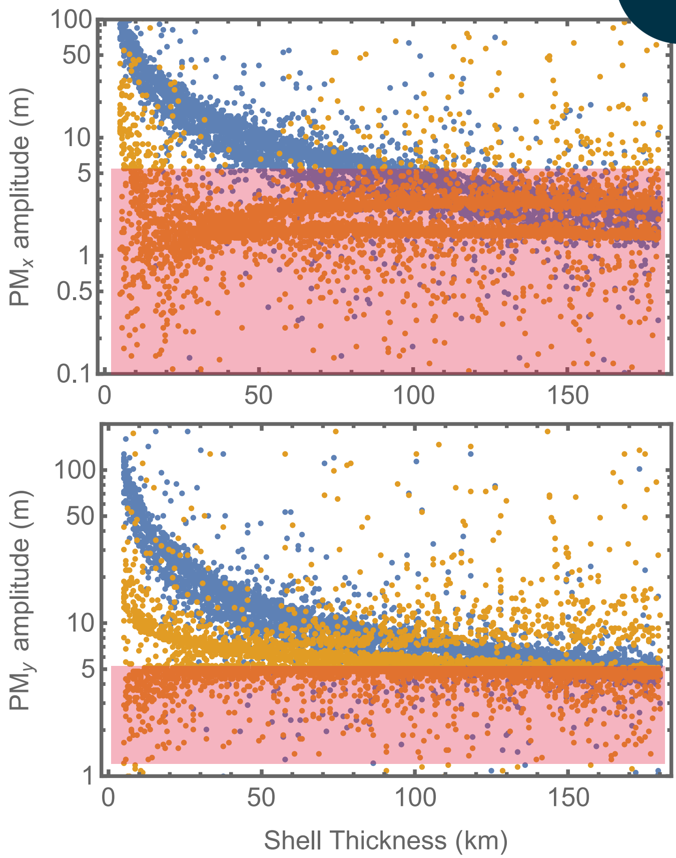
$\frac{d}{dt}\hat{p}_k + \overrightarrow{\Omega}_k \times \hat{p}_k = 0$ for each solid layer ($k = i$ or $k = s$).



Influence of a subsurface ocean

- Possible resonant amplification of the obliquity and the PM
- JUICE obliquity precision : ~0.2 arcsec





Conclusion and perspectives

- Triaxiality of the icy satellites influences the CS

- Presence of a subsurface ocean can lead to resonant amplification of the obliquity and PM (constraints of the interior by JUICE ?)

- Influence of the tidal deformations on Cassini States still need to be studied

Thank you for your attention !