l'Observatoire de Paris | SYRTE

Systèmes de Référence Temps-Espace

Radio science simulations in General Relativity and in alternative theories of gravity

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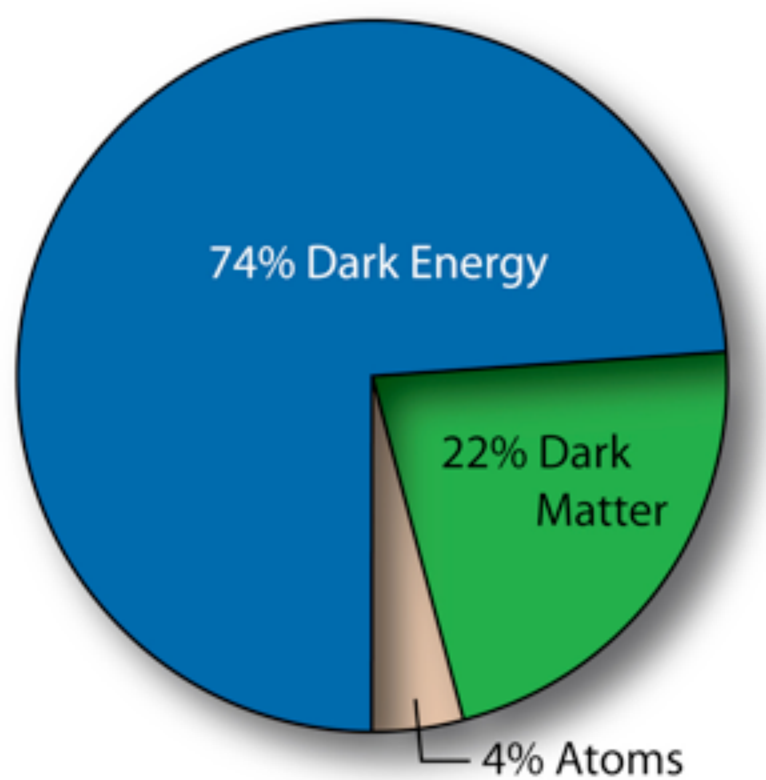
Motivations

- General Relativity (GR) is very well tested in the solar system (light deflection, radio science experiments, ephemerides).
- But... still a lot of interest to perform test of General Relativity:
  - theoretical problem: GR is not the ultimate theory of gravity: quantum theory of gravity, unification with other interactions
  - cosmological problem: no direct detection of Dark Matter and Dark Energy → alternative theory of gravity to explain this kind of effects



Idea of the work

- Test of GR at solar system scale with space mission: radio science experiments
- New tool that performs Range/Doppler simulations from a specific space-time metric (GR or in alternative theories of gravity) and fits of the orbital initial conditions in GR
- Possible to have quick idea of the order of magnitude/signature of the gravitation theory on Range/Doppler signals
  - order of magnitude of relativistic corrections
  - order of magnitude of expected deviations
  - correlations of these deviations with the initial conditions
- In this presentation: Cassini (between Jupiter and Saturn) in Post-Einsteinian Gravity[1] (PEG) or with "MOND" External Field Effect [2]

[1] M.T. Jaekel, S. Reynaud, Class. and Quantum Grav. 22/2135, 2005

[2] L. Blanchet, J. Novak, MNRAS, 2011

Current tests of GR

- Until now, solar-system tests of GR are based on PPN framework. Metric parametrized with 10 constants parameters (famous γ and β).

$$\begin{aligned} g_{00} &= 1 + 2\phi_N + 2\beta\phi_N^2 \\ g_{ij} &= (-1 + 2\gamma\phi_N)\,\delta_{ij} \end{aligned}$$

with ϕ_N the Newtonian potential

- 30 years of precise experiments have constrained PN parameters very closely around GR

$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$ Doppler during a solar conjunction[1]

$$\beta - 1 = (-6.2 \pm 8.1) \times 10^{-5}$$ INPOP10a ephemerides [2]

- Confirmed by many other experiments: VLBI (light deflection), Lunar Laser Ranging, Mars orbiters, ...

... and expected to be improved in the future (GAIA, BepiColombo around Mercury, ...)

[1] B. Bertotti, L. Iess, P. Tortora, Nature, 425/374, 2003

[2] A. Fienga, Moriond Conference, 2011

Is it necessary to go beyond ?

- Until now, solar-system tests of GR are based on PPN framework.
- Theoretical model predicts PPN deviation of smaller amplitude than current accuracy[1]
- not all theories can enter in the PPN framework !! 2 examples: Post-Einsteinian Gravity and MOND External Field Effect
  - PEG theory[2]: phenomenology based on a quantum theory of gravity (1-loop correction) by considering a non-local Einstein field equation

$$
\begin{aligned}
g_{00} &= [g_{00}]_{GR} + 2\delta\Phi_N(r) \\
g_{rr} &= [g_{rr}]_{GR} + 2\delta\Phi_N(r) - 2\delta\Phi_p(r)
\end{aligned}
$$

  - MOND External Field Effect (EFE)[3]: effect due to boundary conditions - modification of the Newtonian potential

$$\phi = \frac{GM}{r} + \frac{Q_2}{2} x^i x^j \left(e_i e_j - \frac{1}{3}\delta_{ij} \right)$$

- What are the effects of these theories on Range/Doppler signals ? Can they be observed ? Simulations performed directly from metric!

[1] T. Damour, K. Nordvedt, Phys. Rev. D, 48/3436, 1993
[2] M.T. Jaekel, S. Reynaud, Class. and Quantum Grav. 22/2135, 2005
[3] L. Blanchet, J. Novak, MNRAS, 2011

Covariant Range/Doppler

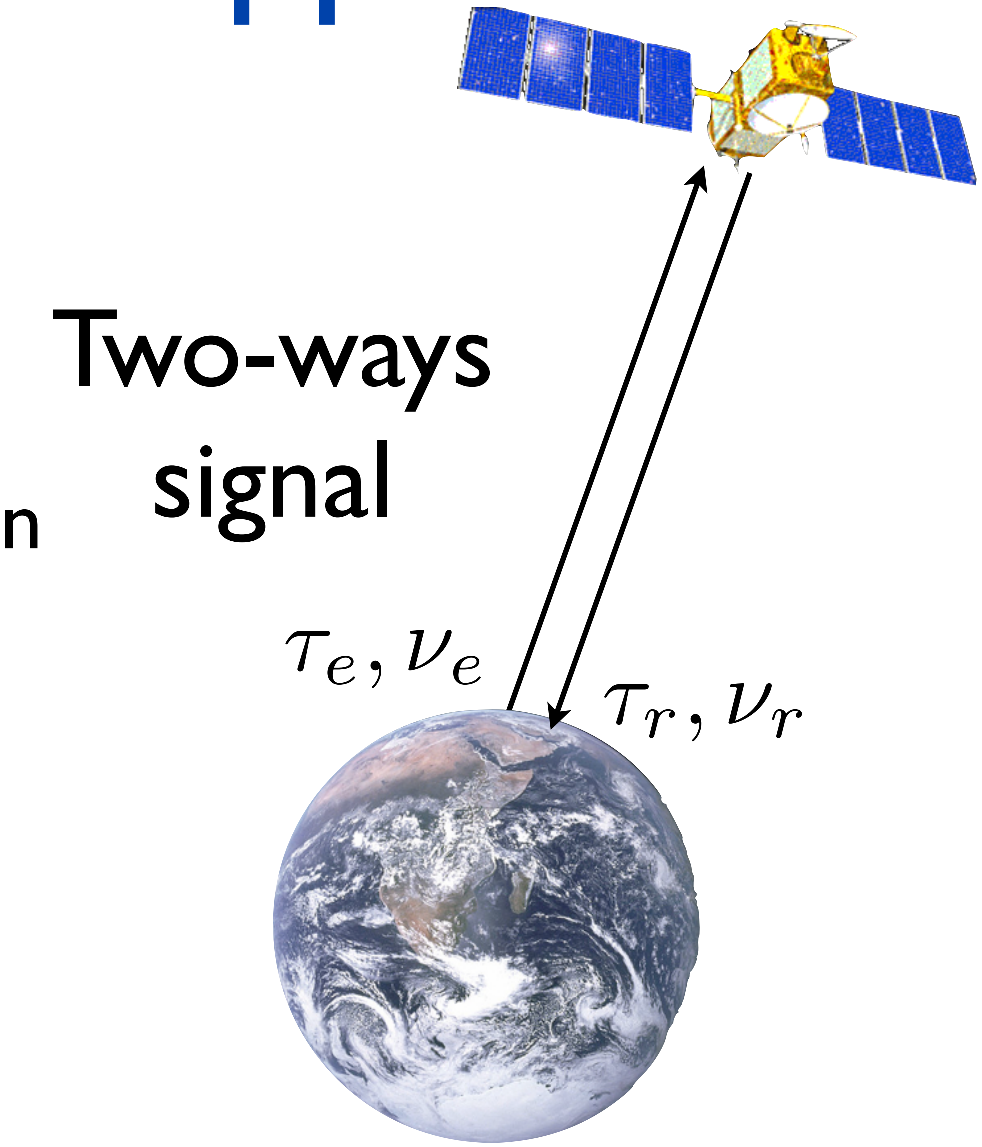
- Range is related to propagation time: difference between receptor proper time and emitter proper time

$$R(\tau_r) = \tau_r - \tau_e$$

- Doppler is proper frequency shift between emission and reception

$$D(\tau_e) = \frac{\nu_r}{\nu_e}$$

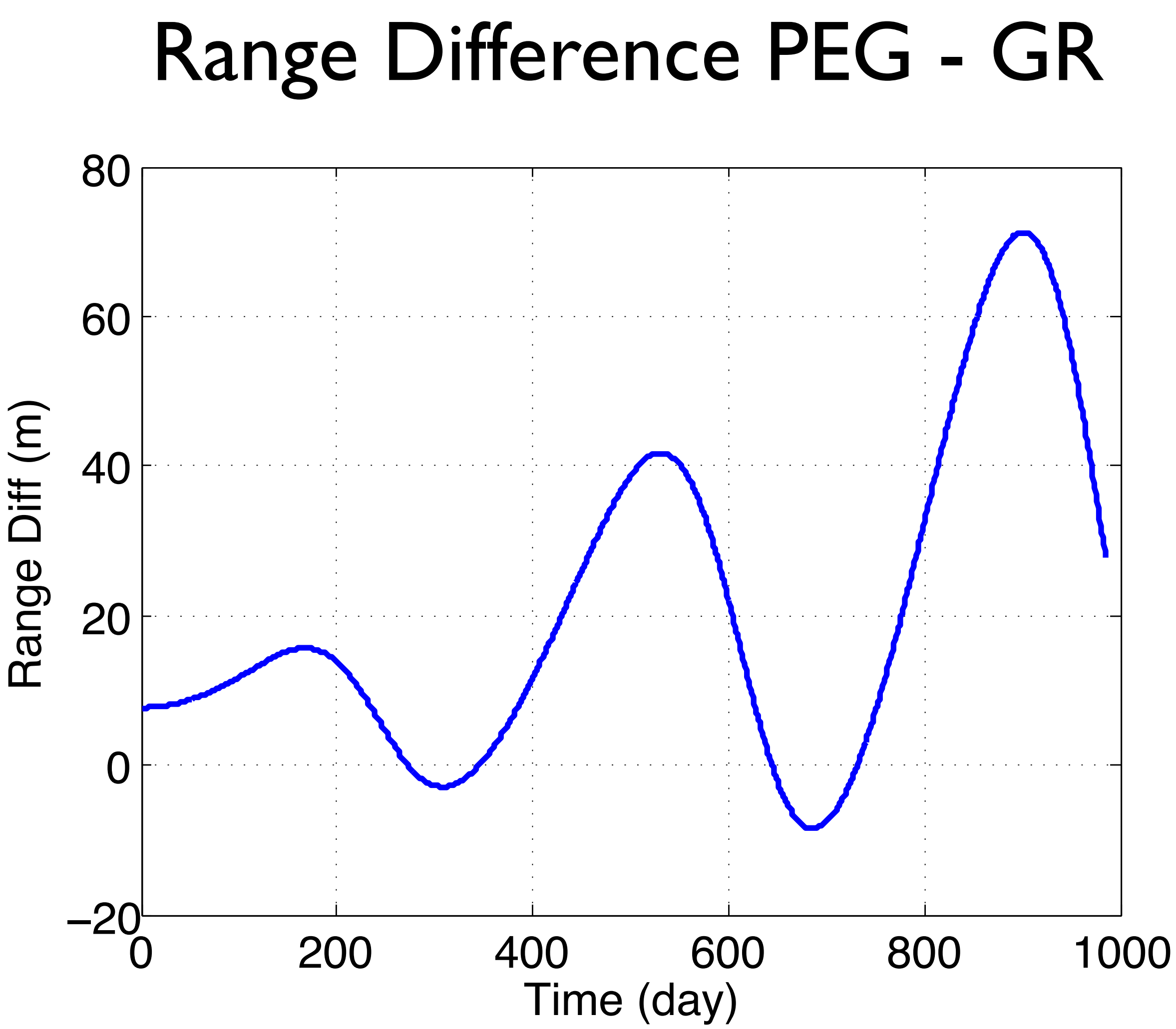
- These definitions are covariant: do not depend on the coordinates system

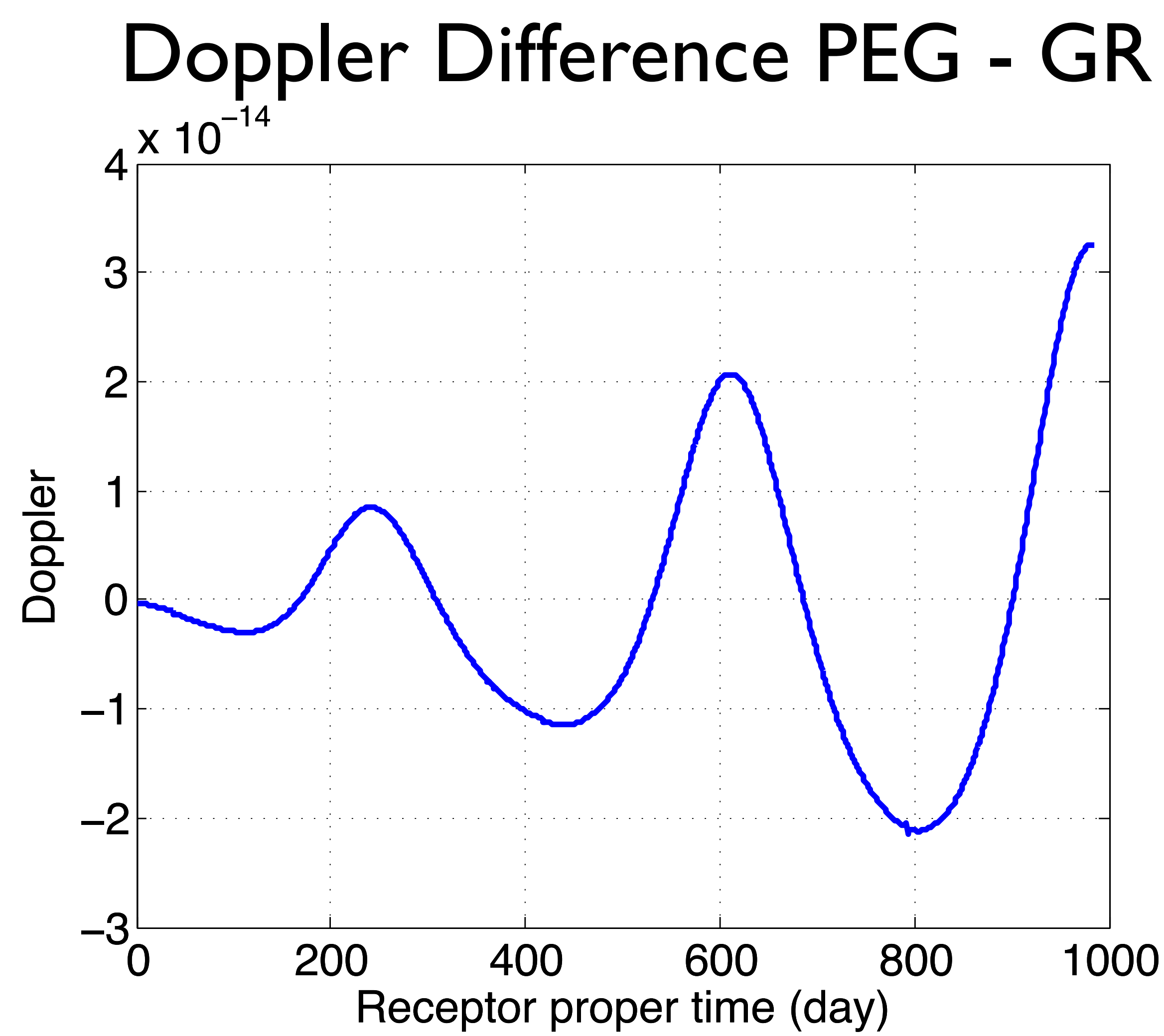


- Simulations: orbit of spacecraft/planets, clock behavior, light propagation directly from space-time metric
- Comparison with GR: fit of the initial conditions needed
  - to avoid effects due to the choice of coordinates
  - this fit is always performed in practice

Example 1: PEG effects on Cassini

- Alternative theory: PEG in the second sector[1]: $g_{rr} = [g_{rr}]_{GR} - 2\delta\Phi_p(r)$ with $\delta\Phi_p(r) = \sum_i \chi_i r^i$
- Cassini Range/Doppler simulations with $\chi_1 = 10^{-23}\ \mathrm{m}^{-1}$.





[1] M.T. Jaekel, S. Reynaud, Class. and Quantum Grav. 22/2135, 2005
M.T. Jaekel, S. Reynaud, Class. and Quantum Grav. 23/777, 2006

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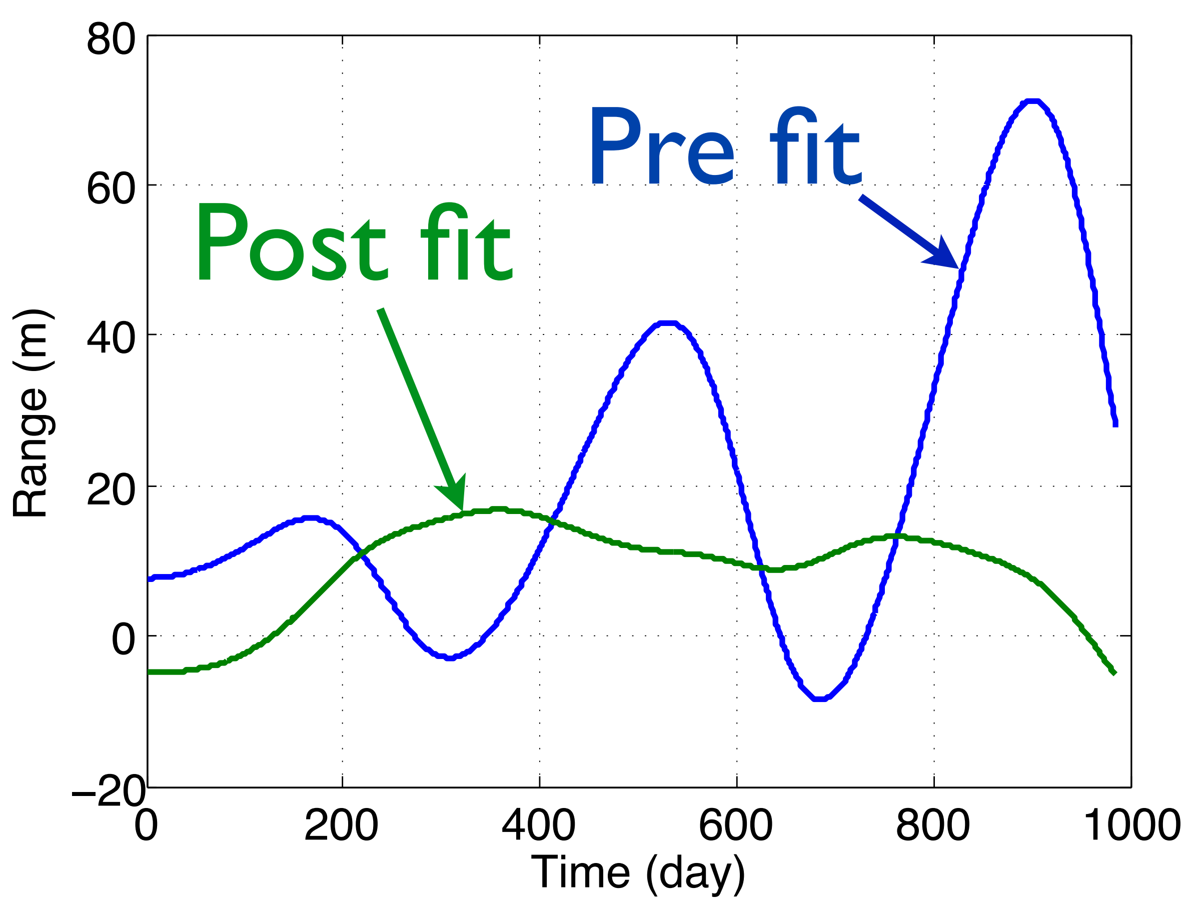
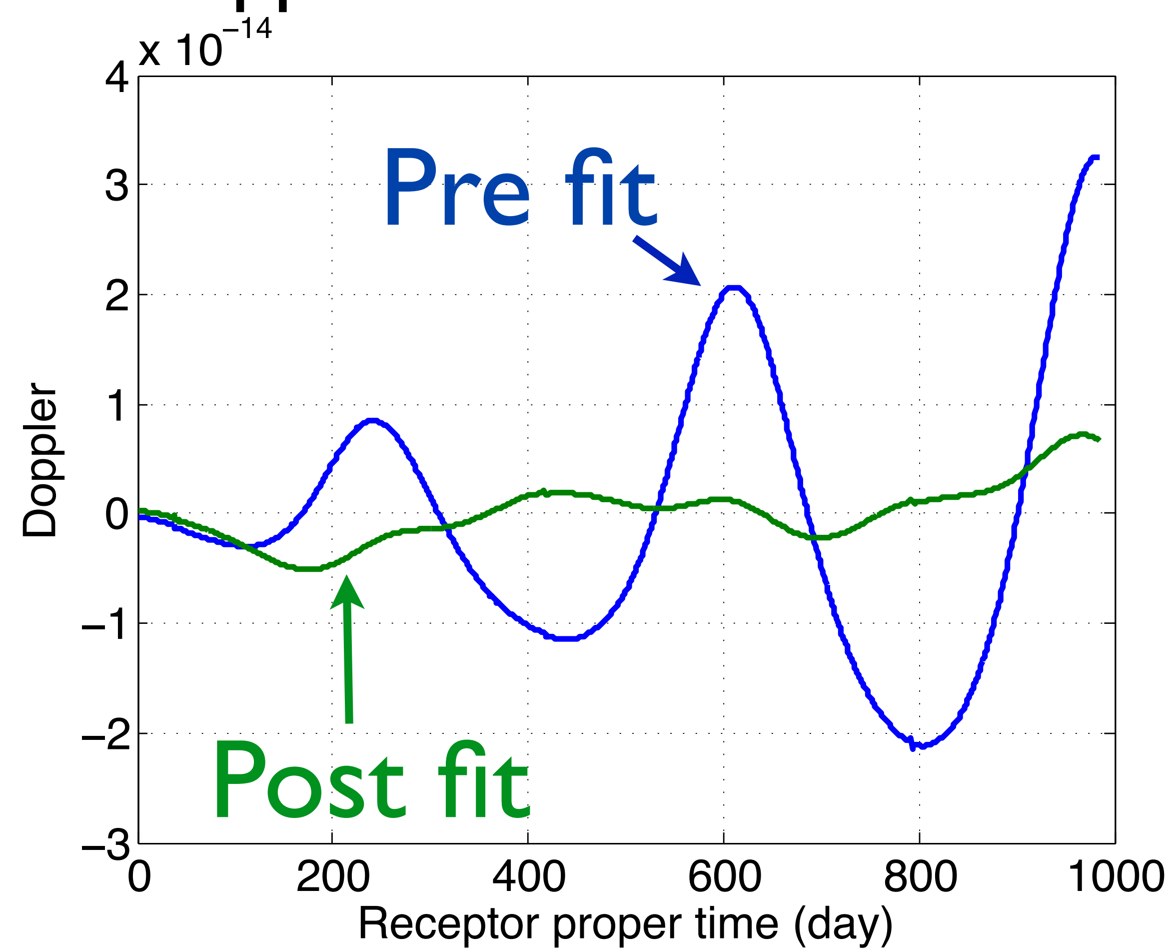
- Fit of Cassini initial conditions in GR

Range Difference PEG - GR



Doppler Difference PEG - GR

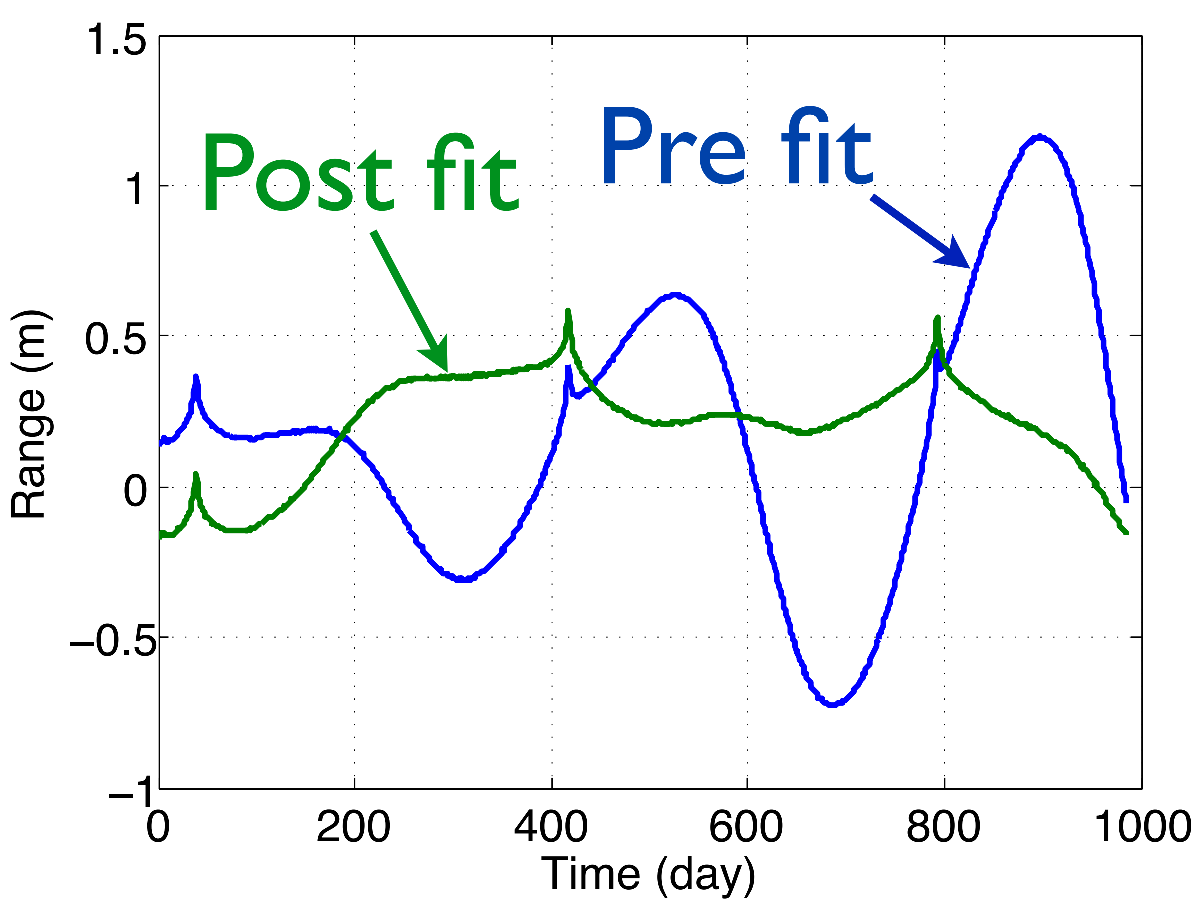


[1] M.T. Jaekel, S. Reynaud, Class. and Quantum Grav. 22/2135, 2005
M.T. Jaekel, S. Reynaud, Class. and Quantum Grav. 23/777, 2006

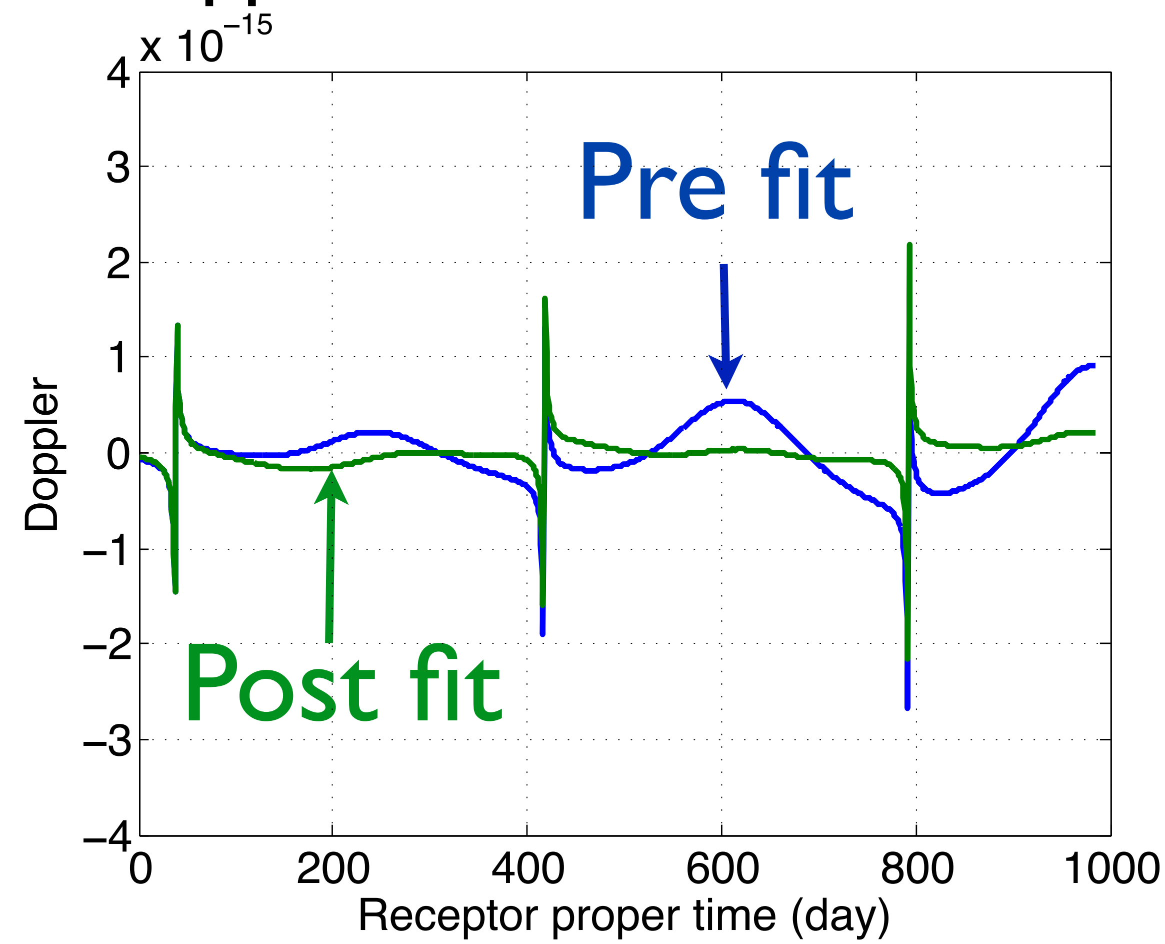
Example 2: PPN effects on Cassini

- Cassini Range/Doppler simulations with $\gamma\text{-}1=10^{-5}$.
- Modification of the metric $g_{rr} = [g_{rr}]_{GR} - 2(\gamma - 1)\frac{GM}{rc^2}$
- Fit of the initial conditions of Cassini in GR

Range Difference PEG - GR



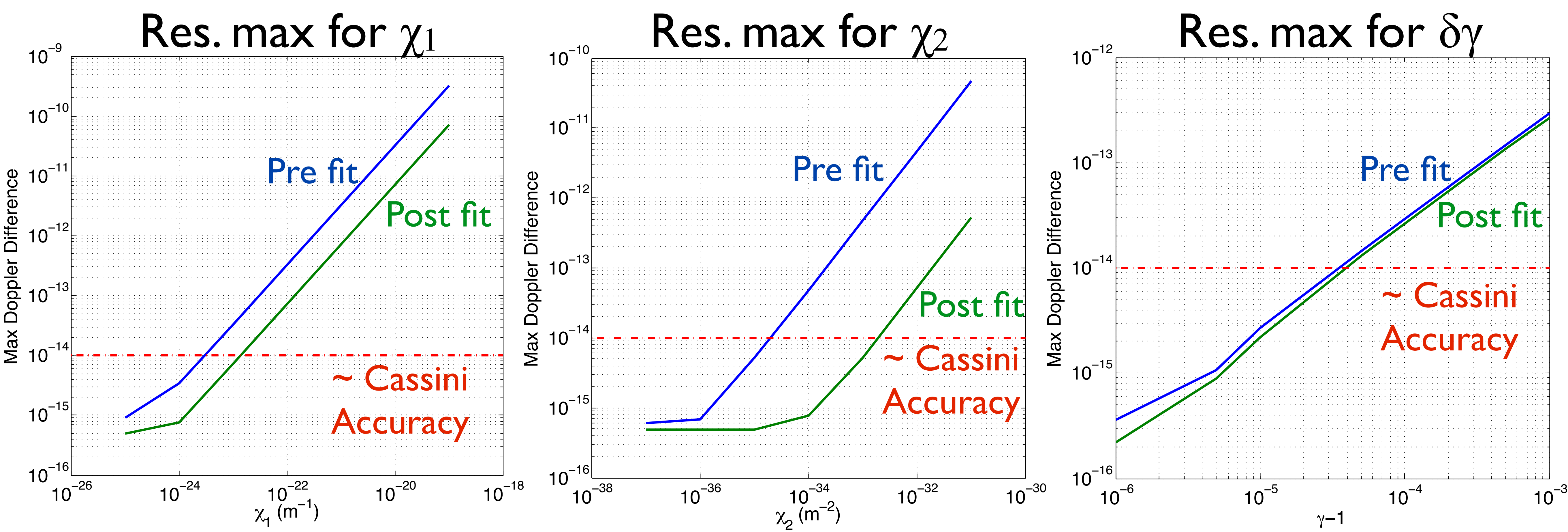
Doppler Difference PEG - GR



Example: 3 PEG parameters

- Cassini Doppler simulations with χ_1, χ_2, $\delta\gamma = \gamma - 1$
- Modification of the metric $g_{rr} = [g_{rr}]_{GR} - 2\chi_1 r - 2\chi_2 r^2 - 2\delta\gamma \frac{GM}{c^2 r}$
- Maximum of the residuals (pre and post-fit) for different values of PEG parameters

- Cassini precision (assumed at $\sim 10^{-14}$ on Doppler) gives an idea of the precision that we can get on parameters: $\chi_1 \sim 10^{-23} \mathrm{m}^{-1}$, $\chi_2 \sim 2\ 10^{-33} \mathrm{m}^{-2}$, $\gamma - 1 \sim 3\ 10^{-5}$ (similar to Bertotti et al precision[1]).



[1] B. Bertotti, L. Iess, P. Tortora, Nature, 425/374, 2003

Example: MOND field

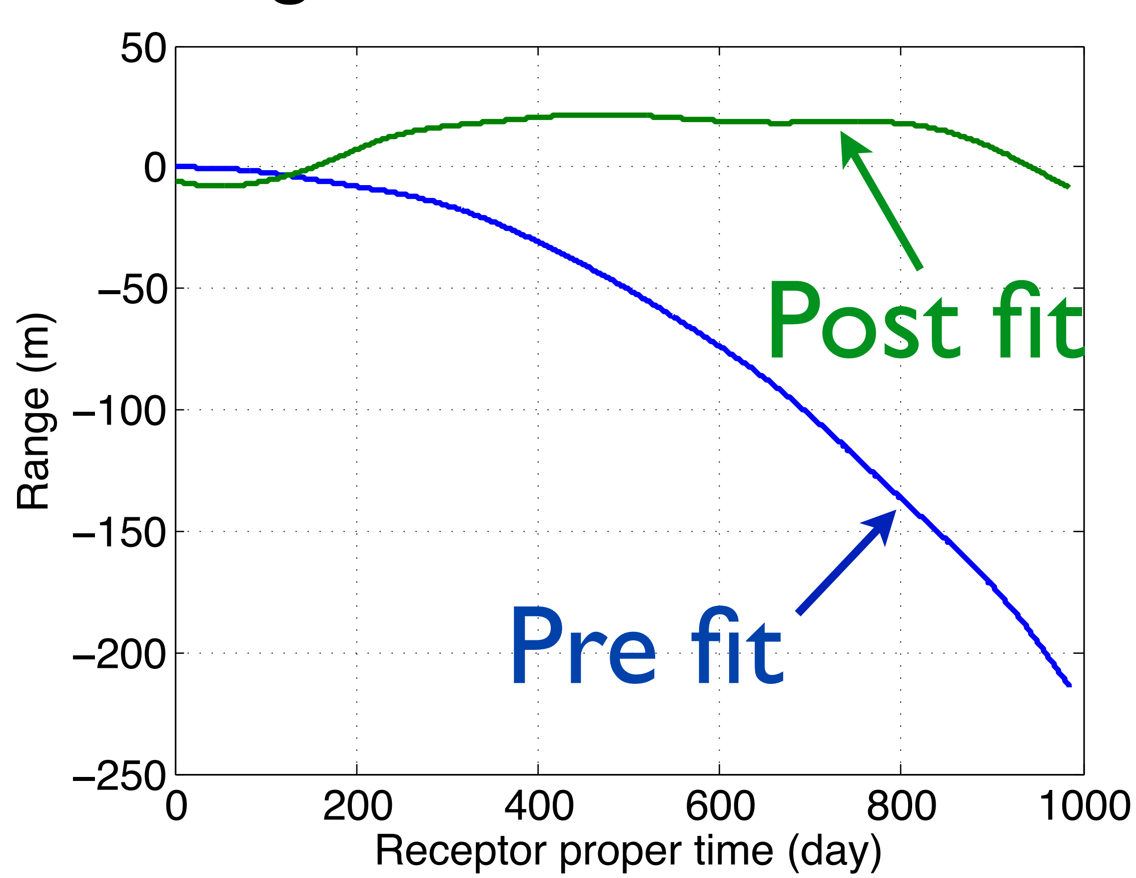
- The dominant effect (External Field Effect) of MOND around Sun is a quadrupole[1]

$$U = \frac{GM}{r} + \frac{Q_2}{2} x^i x^j \left(e_i e_j - \frac{1}{3}\delta_{ij} \right)$$

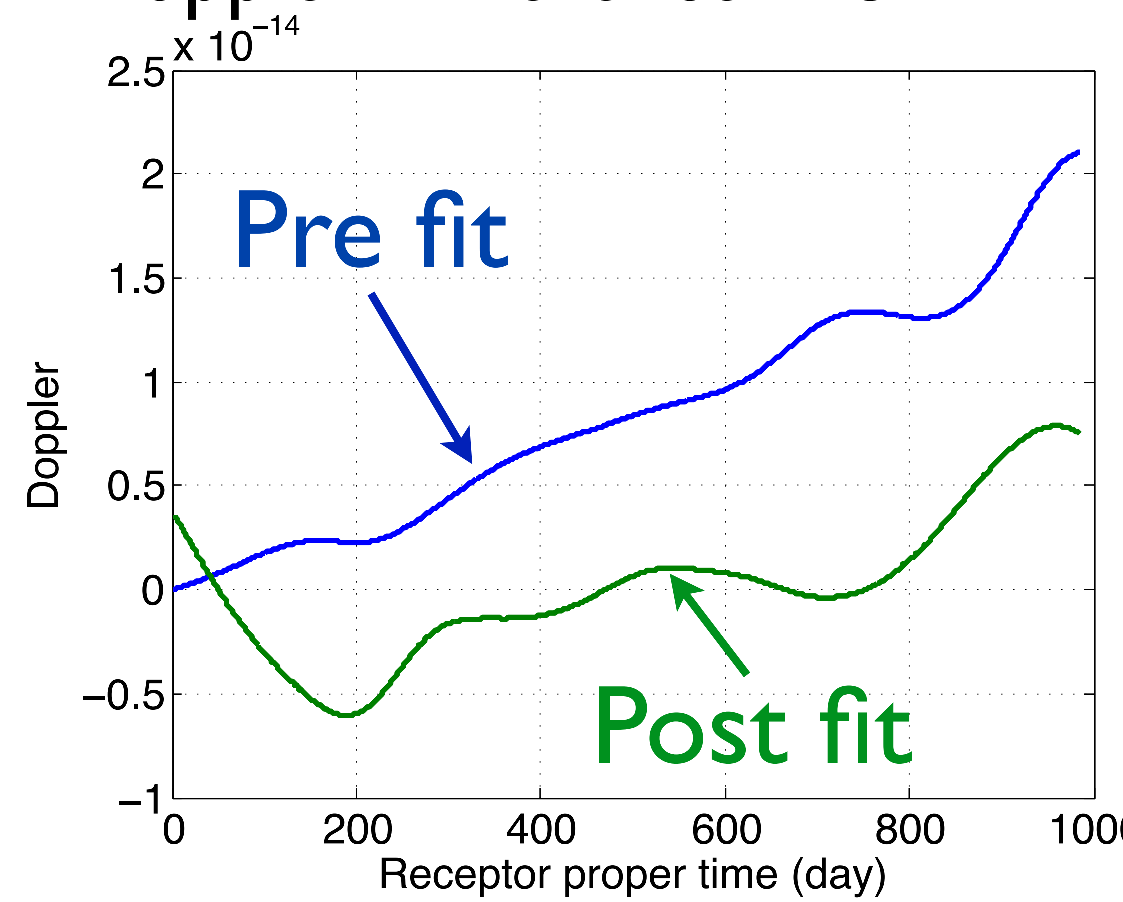
with $2.1\ 10^{-27}\ s^{-2} \leq Q_2 \leq 4.1\ 10^{-26}\ s^{-2}$

- Range/Doppler simulations and fit with upper bound on Q_2
- Signals and residuals below Cassini accuracy: Cassini not useful to test MOND theory

Range Difference MOND - GR



Doppler Difference MOND - GR



[1] L. Blanchet, J. Novak, MNRAS, 2011

Conclusion

- Software that simulates Range/Doppler observables directly from the space-time metric
- Software that performs a fit of the initial conditions in GR
- very easy and very fast to give an idea of the amplitude and signature of a particular deviation from GR on the Range and Doppler of any space mission (even for theory beyond PPN formalism)
- As an example: PPN/PEG simulations were presented on Cassini spacecraft → constraint on PEG parameters derived
- Other Ex.: MOND theory signature on Cassini just too small to be detected. Other space mission?

Numerical simulations were made on the local computing resources (cluster URBM-SysDyn) at the University of Namur (FUNDP)

Perspectives

- perform a lot of simulations with different gravitation theories on different (future and past) space missions

  Answer the question: can a particular deviation from GR be seen with a selected space mission ?

- include more effects to predict more subtle correlations: asteroid belt, planetary gravitational field, non-gravitational forces on spacecraft...

- extend the work to the another type of measurement done in the solar system: direction of light ray (VLBI)

Numerical simulations were made on the local computing resources (cluster URBM-SysDyn) at the University of Namur (FUNDP)