FNRS Contact Group Meeting

On the stability of 3D exoplanetary systems

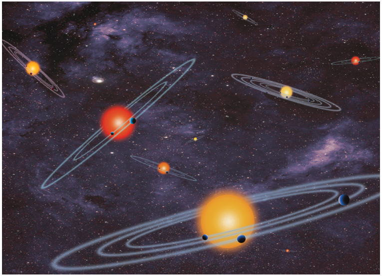
Mara Volpi

Joint work with Anne-Sophie Libert and Arnaud Roisin

Université de Namur

naxys
Namur Institute for Complex Systems

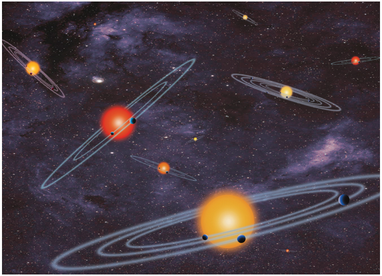
Motivation: a Partial Knowledge



Current data ([exoplanets.eu]:

- 3781 planets
- 2829 systems, 629 multi-planetary

Motivation: a Partial Knowledge

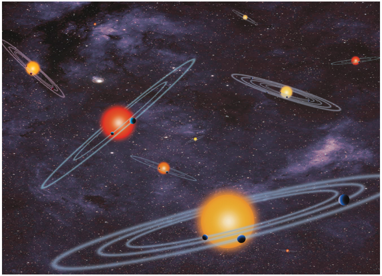


Current data ([exoplanets.eu]:

- 3781 planets
- 2829 systems, 629 multi-planetary

As for the exact structure...

Motivation: a Partial Knowledge



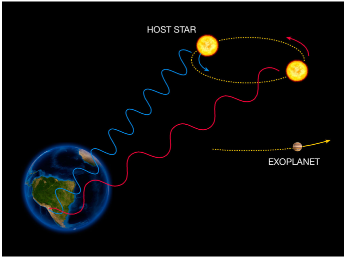
Current data ([exoplanets.eu]:

- 3781 planets
- 2829 systems, 629 multi-planetary

As for the exact structure...

... we have no idea!

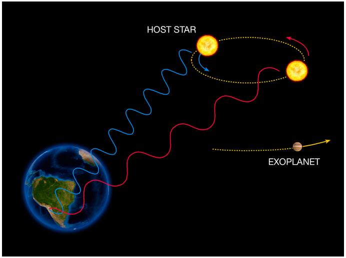
LIMITATIONS OF THE DETECTION METHODS



RV-detected giant planetary systems

- no information about the **inclinations**
- only **minimal masses** $\frac{m}{\sin(i)}$

LIMITATIONS OF THE DETECTION METHODS



RV-detected giant planetary systems

- no information about the **inclinations**
- only **minimal masses** $\frac{m}{\sin(i)}$

GOAL

Identify possible 3D architecture of RV-detected systems

Analytical Work Environment

- Two-planet systems (gravitational forces)
- Laplace plane: constant reference plane
- Secular Approach: far from mean motion resonances

Analytical Work Environment

- Two-planet systems (gravitational forces)
- LAPLACE PLANE: constant reference plane
- Secular Approach: far from mean motion resonances

$$\Downarrow$$

Only 2 degrees of freedom

- Two-planet systems (gravitational forces)
- LAPLACE PLANE: constant reference plane
- Secular Approach: far from mean motion resonances

$\Downarrow$

Only 2 degrees of freedom

$\swarrow$

Nice thing #1

Less sensitivity to initial conditions

- Two-planet systems (gravitational forces)
- LAPLACE PLANE: constant reference plane
- Secular Approach: far from mean motion resonances

$\Downarrow$

Only 2 degrees of freedom

$\swarrow \quad \searrow$

Nice thing #1

Less sensitivity to initial conditions

Nice thing #2

Faster integrations

Analytical Work Environment

- Two-planet systems (gravitational forces)
- LAPLACE PLANE: constant reference plane
- Secular Approach: far from mean motion resonances

$\Downarrow$

Only 2 degrees of freedom

$\swarrow \quad \searrow$

Nice thing #1

Less sensitivity to initial conditions

Nice thing #2

Faster integrations

First order in the masses **Secular Hamiltonian**

$$H(D_2, \boldsymbol{\xi}, \boldsymbol{\eta}) = \sum_{j=0}^{\mathrm{ORDECC}/2} \mathcal{C}_{j,\boldsymbol{m},\boldsymbol{n}} \, D_2^j \sum_{|\boldsymbol{m}|+|\boldsymbol{n}|=0}^{\mathrm{ORDECC}-j} \boldsymbol{\xi}^{\boldsymbol{m}} \boldsymbol{\eta}^{\boldsymbol{n}}$$

where $\boldsymbol{\xi}, \boldsymbol{\eta}$ are heliocentric canonical Poincaré variables and D_2 is the Angular Momentum Deficit [Robutel 1995, Laskar 1997]

Importance of Lidov-Kozai resonance for 3D configurations

Lidov-Kozai resonance [Lidov 1961, Kozai 1962]

PROTECTIVE MECHANISM FOR HIGLY INCLINED SYSTEMS

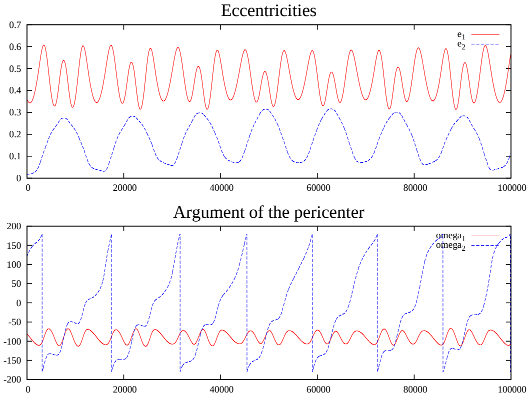
- Typically appears at $\simeq 40^\circ$
  (depending on mass and semi-major axis ratios [Libert & Henrard 2007])
- In the Laplace plane, argument of pericenter ω_1 librates around $\pm 90^\circ$

Importance of Lidov-Kozai resonance for 3D configurations

Lidov-Kozai resonance [Lidov 1961, Kozai 1962]

PROTECTIVE MECHANISM FOR HIGLY INCLINED SYSTEMS

- Typically appears at $\simeq 40^\circ$
  (depending on mass and semi-major axis ratios [Libert & Henrard 2007])
- In the Laplace plane, argument of pericenter ω_1 librates around $\pm 90^\circ$



Objectives

THE TOOL

Secular Approximation
Hamiltonian

Objectives

The Tool

Secular Approximation
Hamiltonian

↙

Question 1

Detection of Lidov-Kozai
resonance regions?

Objectives

The Tool

Secular Approximation
Hamiltonian

↙ ↘

Question 1

Detection of Lidov-Kozai
resonance regions?

Question 2

Possible stable
3D configurations?

The Systems

- Two planets
- Non resonant
- masses $< 10 M_J$
- periods > 45 days
- semi-major axis < 10 AU

Fixed parameters

All the observed orbital parameters ([exoplanets.eu]):

- semi-major axes
- eccentricities
- arguments of the pericenter

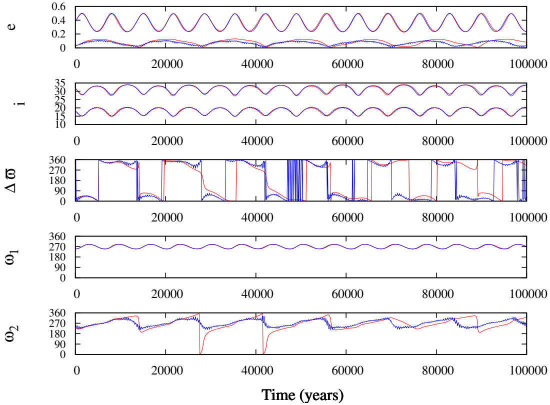
Hypothesis

Same inclination of the orbital planes, $i_1 = i_2 = i$

Changing parameters

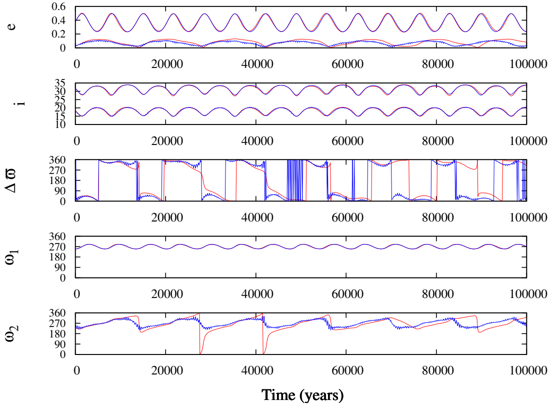
- i from 5° to 90°
- masses change: multiplying factor $1/\sin(i)$
- i_{mut} from 0° to 80°

HD 12661: $i = 50$, $i_{mut} = 50^{\circ}$



Validation of the analytical approach

HD 12661: $i = 50$, $i_{mut} = 50^\circ$



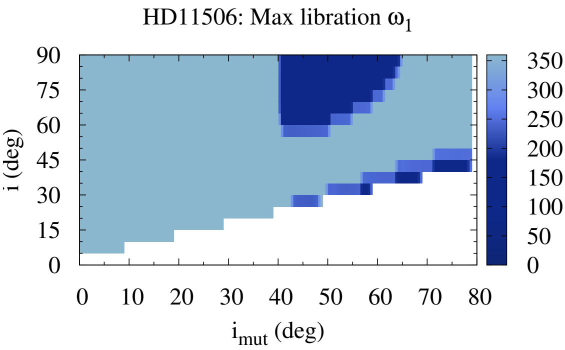
Very good agreement with numerical integrations up to very high mutual inclination

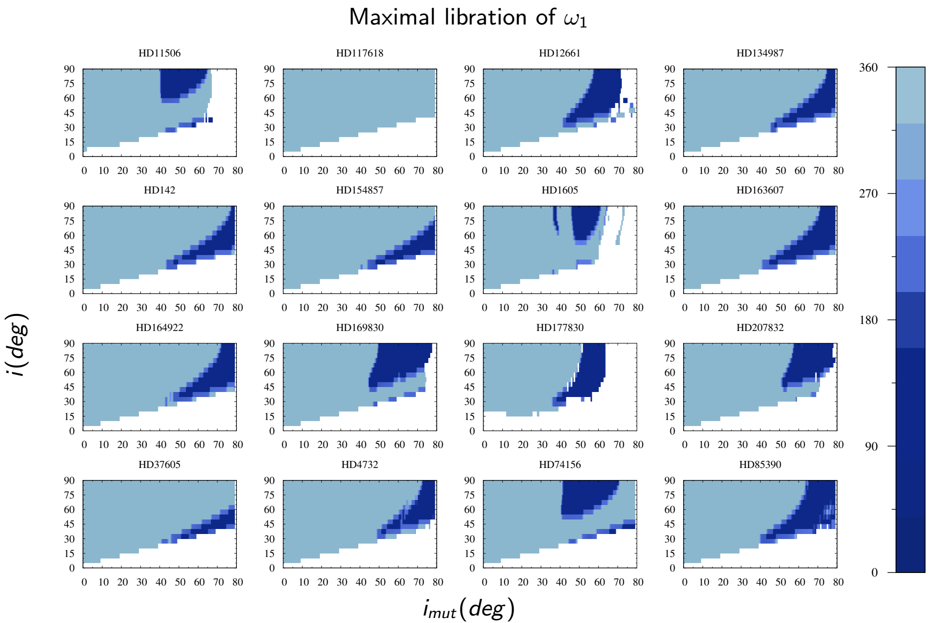
Question 1

Detection of Lidov-Kozai resonance regions?

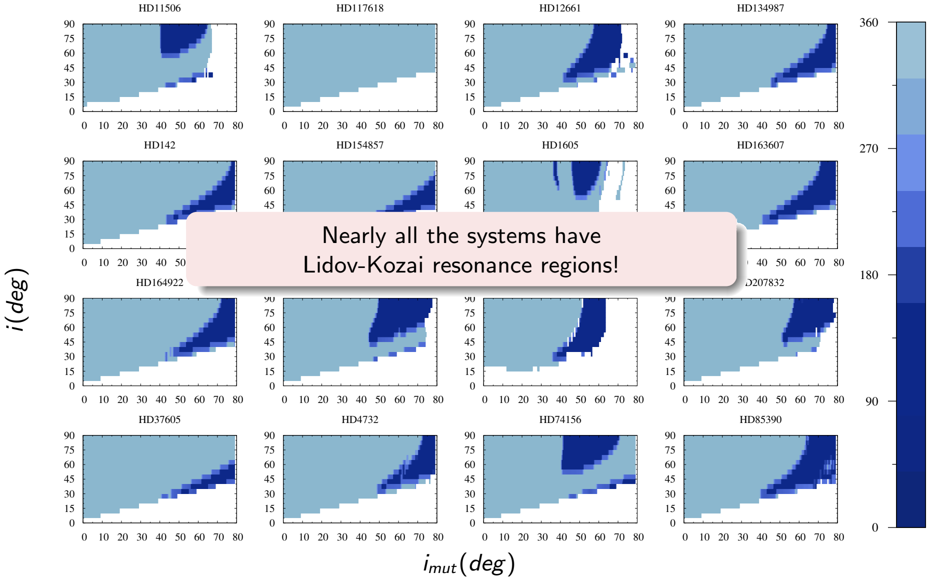
Question 1

Detection of Lidov-Kozai resonance regions?




Maximal libration of ω_1
HD11506
HD117618
HD12661
HD134987
HD142
HD154857
HD1605
HD163607
HD164922
HD169830
HD177830
HD207832
HD37605
HD4732
HD74156
HD85390
$i(deg)$
$i_{mut}(deg)$
0
90
180
270
360

Maximal libration of ω_1



Nearly all the systems have
Lidov-Kozai resonance regions!

System	min i_{mut} (°)	min i (°)	Lidov-Kozai (%)
HD 11506	40.5	30	17
HD 117618	–	–	–
HD 12661	42.5	30	20
HD 134987	45.5	40	13
HD 142	44	30	11
HD 154857	40.5	30	10
HD 1605	36	20	12
HD 163607	41	30	16
HD 164922	43	30	14
HD 169830	45	25	22
HD 177830	48.5	25	21
HD 207832	50	35	17
HD 37605	41	25	7
HD 4732	49	35	12
HD 74156	41	30	20
HD 85390	40	30	21

Table: Summary of the results for all the systems. Minimum values of i_{mut} and i (in degrees) for which we register the libration of the angle ω_1; percentage of points that show a Lidov-Kozai resonance; percentage of points classified as chaotic by the MEGNO.

Possible stable 3D configurations

Question 2

Possible stable 3D configurations?

Parametric study through chaos indicator: MEGNO

Let $H(\boldsymbol{p}, \boldsymbol{q})$ with $\boldsymbol{p}, \boldsymbol{q} \in \mathbb{R}^N$ be an autonomous Hamiltonian of N degrees of freedom. Being $\boldsymbol{x} = (\boldsymbol{p}, \boldsymbol{q}) \in \mathbb{R}^{2N}$, the Hamiltonian vector field can be expressed as

$$\dot{\boldsymbol{x}} = J \nabla_{\boldsymbol{x}} \mathcal{H} \boldsymbol{x} \,.$$

Deviation vectors $\boldsymbol{\delta}(t)$ satisfy the variational equations $\dot{\boldsymbol{\delta}}(t) = J \nabla^2_{\boldsymbol{x}} \mathcal{H} \boldsymbol{\delta}(t)$, being $\nabla^2_{\boldsymbol{x}} \mathcal{H}$ the Hessian matrix of the Hamiltonian.

MEGNO

$$Y(t) = \frac{2}{t} \int_0^t \frac{\dot{\delta}(s)}{\delta(s)} \, ds$$

$\Longrightarrow$

Mean MEGNO

$$\bar{Y}(t) = \frac{1}{t} \int_0^t Y(s) \, ds.$$

The limit for $t \to \infty$ provides a good characterization of the orbits. In fact, we have

- $lim_{t \to \infty} \bar{Y}(t) = 0$ for stable periodic orbits,
- $lim_{t \to \infty} \bar{Y}(t) = 2$ for quasi-periodic orbits and for orbits close to stable periodic ones,
- for irregular orbits, $\bar{Y}(t)$ diverges with time.

Parametric study through chaos indicator: MEGNO

Let $H(\boldsymbol{p}, \boldsymbol{q})$ with $\boldsymbol{p}, \boldsymbol{q} \in \mathbb{R}^N$ be an autonomous Hamiltonian of N degrees of freedom. Being $\boldsymbol{x} = (\boldsymbol{p}, \boldsymbol{q}) \in \mathbb{R}^{2N}$, the Hamiltonian vector field can be expressed as

$$\dot{\boldsymbol{x}} = J\nabla_{\boldsymbol{x}}\mathcal{H}\boldsymbol{x}\,.$$

Deviation vectors $\boldsymbol{\delta}(t)$ satisfy the variational equations $\dot{\boldsymbol{\delta}}(t) = J\nabla^2_{\boldsymbol{x}}\mathcal{H}\boldsymbol{\delta}(t)$, being $\nabla^2_{\boldsymbol{x}}\mathcal{H}$ the Hessian matrix of the Hamiltonian.

MEGNO

$$Y(t) = \frac{2}{t}\int_0^t \frac{\dot{\delta}(s)}{\delta(s)}\, ds$$

$\Longrightarrow$

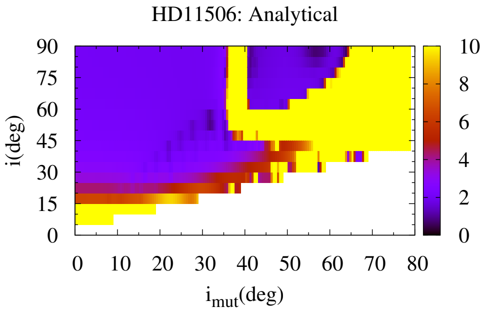
Mean MEGNO

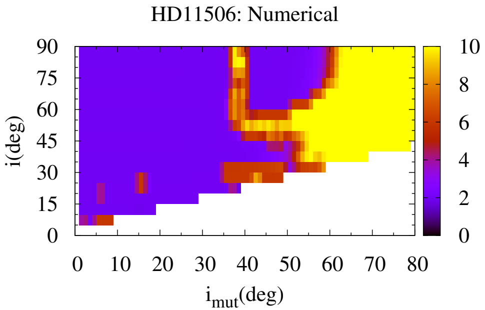
$$\bar{Y}(t) = \frac{1}{t}\int_0^t Y(s)\, ds.$$

The limit for $t \to \infty$ provides a good characterization of the orbits. In fact, we have

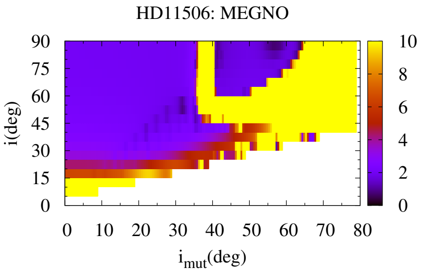
- $lim_{t\to\infty} \bar{Y}(t) = 0$ for stable periodic orbits,
- $lim_{t\to\infty} \bar{Y}(t) = 2$ for quasi-periodic orbits and for orbits close to stable periodic ones,
- for irregular orbits, $\bar{Y}(t)$ diverges with time.

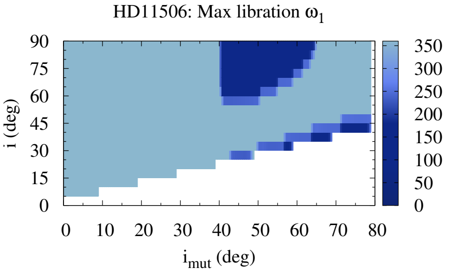
Validation


HD11506: Analytical
90
75
60
45
30
15
0
i(deg)
0
10
20
30
40
50
60
70
80
i_{mut}(deg)
10
8
6
4
2
0


HD11506: Numerical
90
75
60
45
30
15
0
i(deg)
0
10
20
30
40
50
60
70
80
i_{mut}(deg)
10
8
6
4
2
0

MEGNO and 3D stability





Not all values of mutual inclination are compatible with stability for all the systems

MEGNO, generally speaking:

- Stability: low values of i_{mut} and Lidov-Kozai regions
- Chaos: surrounding Lidov-Kozai regions at high values of i_{mut}

We have shown that

- With our secular approximation of the Hamiltonian we study 3D configurations and determine Lidov-Kozai resonance regions
- The systems could be in regular 3D configurations, in most cases up to high mutual inclination i_{mut} (some up to $75^\circ - 80^\circ$!)
- Most of the systems are compatible with significant Lidov-Kozai resonance regions

We have shown that

- With our secular approximation of the Hamiltonian we study 3D configurations and determine Lidov-Kozai resonance regions
- The systems could be in regular 3D configurations, in most cases up to high mutual inclination i_{mut} (some up to $75^\circ - 80^\circ$!)
- Most of the systems are compatible with significant Lidov-Kozai resonance regions

Perspectives

Extend the study to close-in planets

include General Relativity $\rightarrow$ how Lidov-Kozai regions change?

Thank you for your attention!